Periodic meshes for the CGAL library

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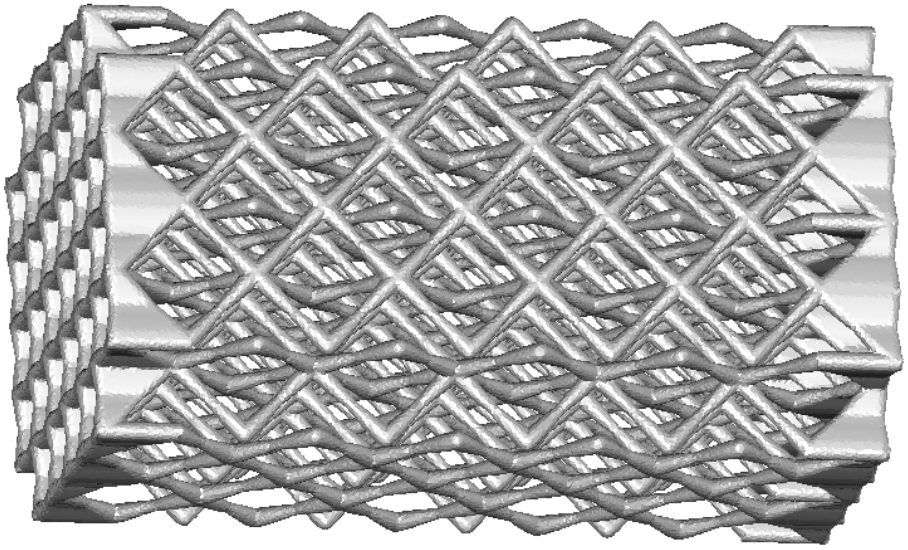
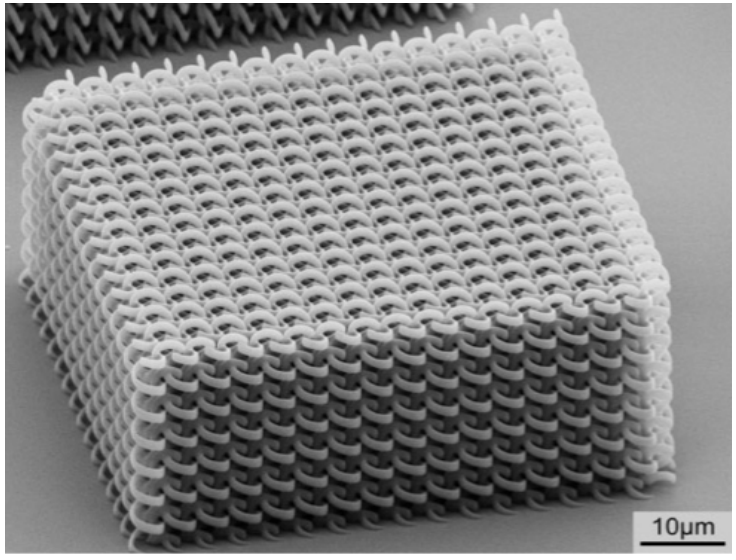
Computational geometry in non-Euclidean spaces
Nancy, August 2015

Motivation: applications

- Material engineering
- Nano-structures
- …

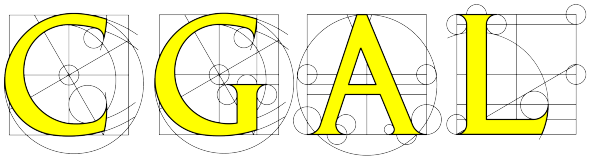


bone scaffolding
M. Moesen, K.U. Leuven



photonic crystal
M. Blome, Zuse Institut Berlin

Existing tools

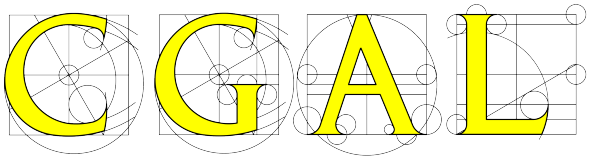


Computational Geometry Algorithms Library

www.cgal.org

- Open source, GPL (+ commercial licences through GEOMETRYFACTORY)
- Generic (C++ templates)
- Robust (“Exact Geometric Computation”)
- Efficient (arithmetic filtering)

Existing tools



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- Generic (C++ templates)
- Robust (“Exact Geometric Computation”)
- Efficient (arithmetic filtering)
- Large variety of packages, in particular
  - 3D **periodic** triangulations
  - 3D **mesh** generation

CGAL 3D periodic triangulations

triangulations in the 3D flat torus $\mathbb{T}^3 = \mathbb{R}^3/\mathcal{G}, \quad \mathcal{G} =< t_x, t_y, t_z >$

$\mathcal{P}$ set of n points in the fundamental domain

Delaunay triangulation defined by $\mathcal{P}$

- defined as a simplicial complex
  no 1- or 2- cycles in graph of edges

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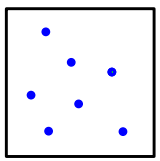

(2D)

$\mathcal{G} =< t_x, t_y >$

$\mathbb{T}^2 = \mathbb{R}^2/\mathcal{G}$

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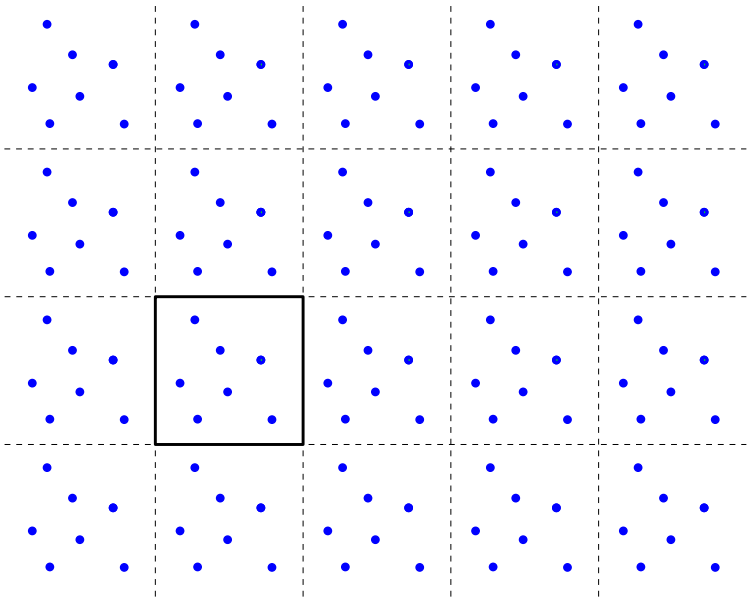

$\mathcal{GP}$

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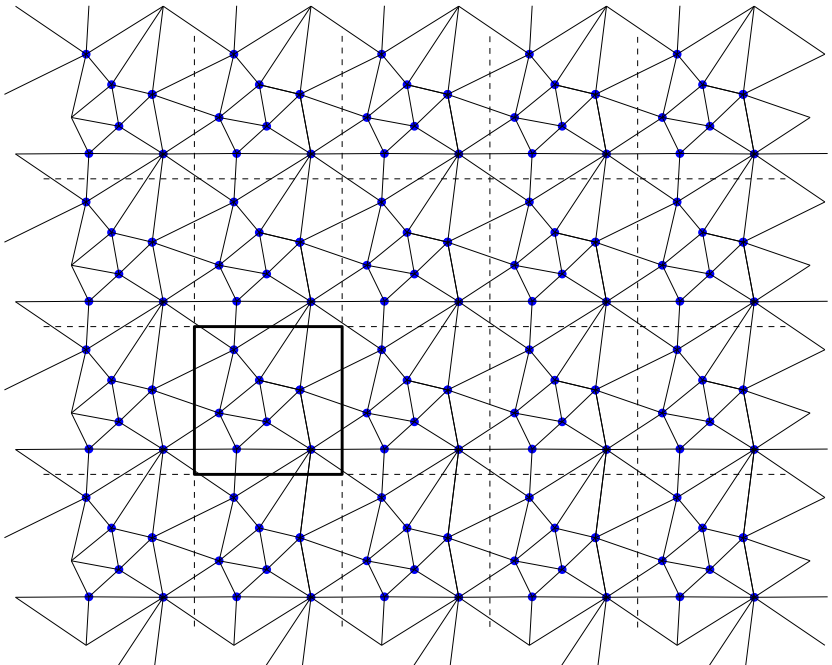

$DT(\mathcal{GP})$

CGAL 3D periodic triangulations

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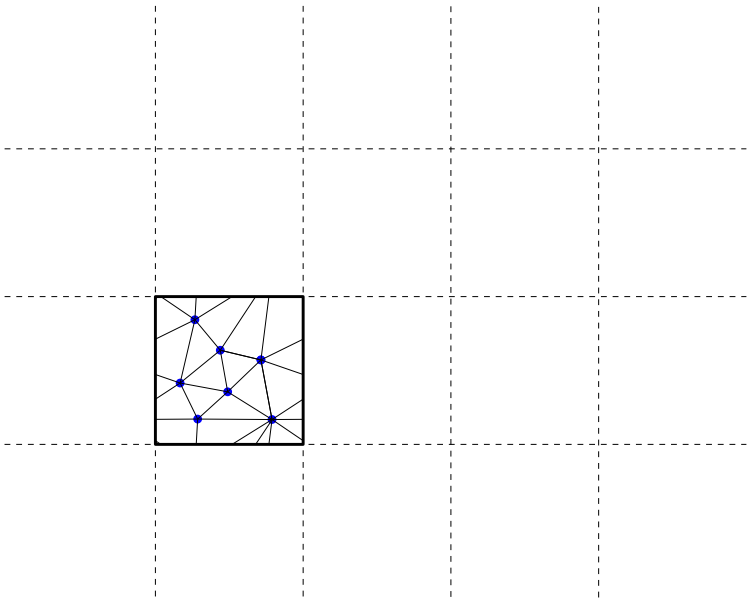


$\mathbb{T}^2 = \mathbb{R}^2/\mathcal{G}$
$\pi : \mathbb{R}^2 \to \mathbb{T}^2$

$DT_{\mathbb{T}^2}(\mathcal{P}) = \pi(DT(\mathcal{GP}))$
if it is a simplicial complex

CGAL 3D periodic triangulations

triangulations in the 3D flat torus $\mathbb{T}^3 = \mathbb{R}^3/\mathcal{G}, \quad \mathcal{G} =< t_x, t_y, t_z >$

Incremental algorithm

- starts in 27-sheeted covering space $\mathbb{R}^3/\mathcal{G}_3$,
$$\mathcal{G}_3 =< 3 \cdot t_x, 3 \cdot t_y, 3 \cdot t_z >$$
- computation in $\mathbb{T}^3$ as soon as sufficient condition on empty ball diameters is satisfied ($< \mathit{cube\_size}/2$)

$\longrightarrow$ randomized worst-case optimal algorithm

$\longrightarrow$ generalizes to general closed Euclidean $d-$manifolds

[M. Caroli & M. T., ESA'09, SoCG'11]

CGAL 3D periodic triangulations

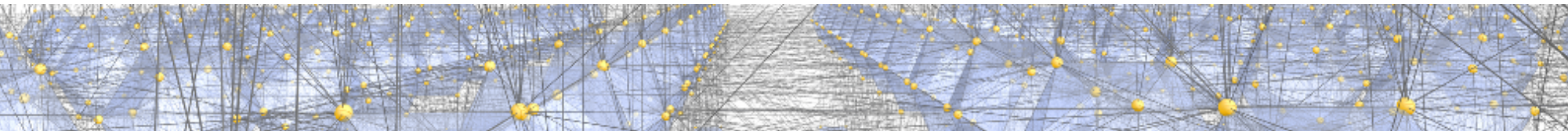
triangulations in the 3D flat torus $\mathbb{T}^3 = \mathbb{R}^3/\mathcal{G}, \quad \mathcal{G} =< t_x, t_y, t_z >$

Periodic Delaunay triangulation package

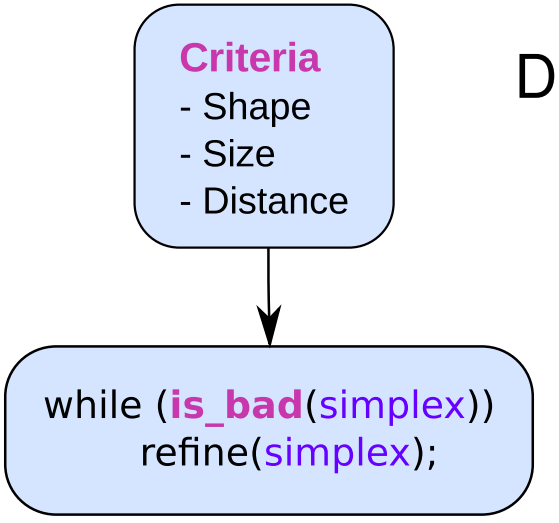
- fully dynamic (insertion/removal)
- all degeneracies handled
- copies of input points *only if needed*
  (avoided in practice)
- running time $\simeq$ 10 million points in 13 sec
  (only $\simeq$ 30% overhead with respect to
  CGAL non-periodic Delaunay triangulations)
- users in various fields

[M. Caroli & M. T., CGAL 3.5, 2009]

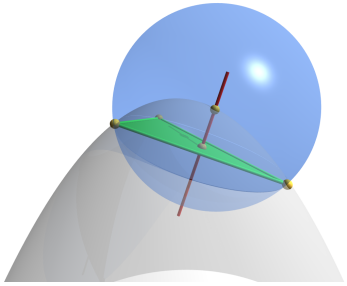
2D [N. Kruithof, CGAL 4.3, 2013]



CGAL 3D volume mesh generation



Delaunay Refinement



Restricted Delaunay triangulation

- flexible: oracle
  surface known through intersection with segment
- input: closed triangulated surface
  output: 722,018 tetrahedra in 66.7s.
- multi-core in CGAL 4.5

[Alliez, Jamin, Rineau, Tayeb, Tournois, Yvinec]

Periodic mesh generation: difficulties and solutions

CGAL 3D volume mesh generation
designed on top of CGAL 3D (non periodic) triangulations

Periodic mesh generation: difficulties and solutions

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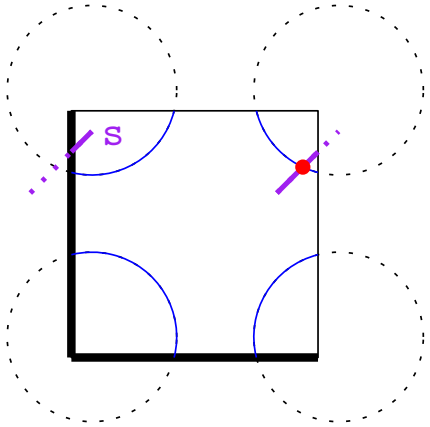
Interface with the 3D periodic triangulations package, *e.g.*

- a vertex is associated with several points
  $\Rightarrow$ modify CGAL code `v→point()` $\mapsto$ `t.point(v)`
- periodic criteria need more information to access points
  $\Rightarrow$ additional template parameter

Periodic mesh generation: difficulties and solutions

CGAL 3D volume mesh generation
designed on top of CGAL 3D (non periodic) triangulations

Semantics of periodic oracle and criteria, *e.g.*
for surface $\mathcal{S}$, `compute_intersection(Segment s)`



there are cases for which
- `s` does not intersect $\mathcal{S}$ in the domain
- a translated copy intersects $\mathcal{S}$ in the domain

$\Rightarrow$ If first call does not find an intersection,
then call again with appropriate translated image

Periodic mesh generation: difficulties and solutions

CGAL 3D volume mesh generation
designed on top of CGAL 3D (non periodic) triangulations

Requires periodic **weighted** Delaunay triangulations for

- optimizations
- handling sharp features

- Also needed by users (without meshes)

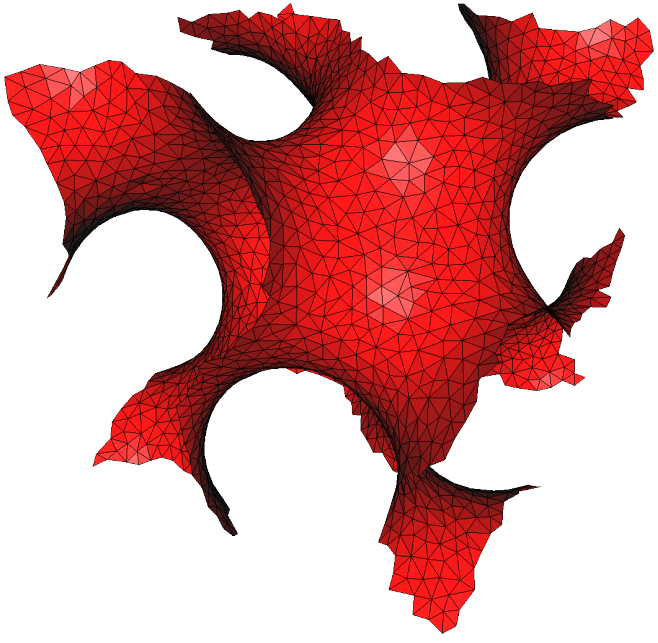
talks by M. Schindler and J. Hiddings

- $\simeq$ ready for integration into CGAL 4.8 (2016)

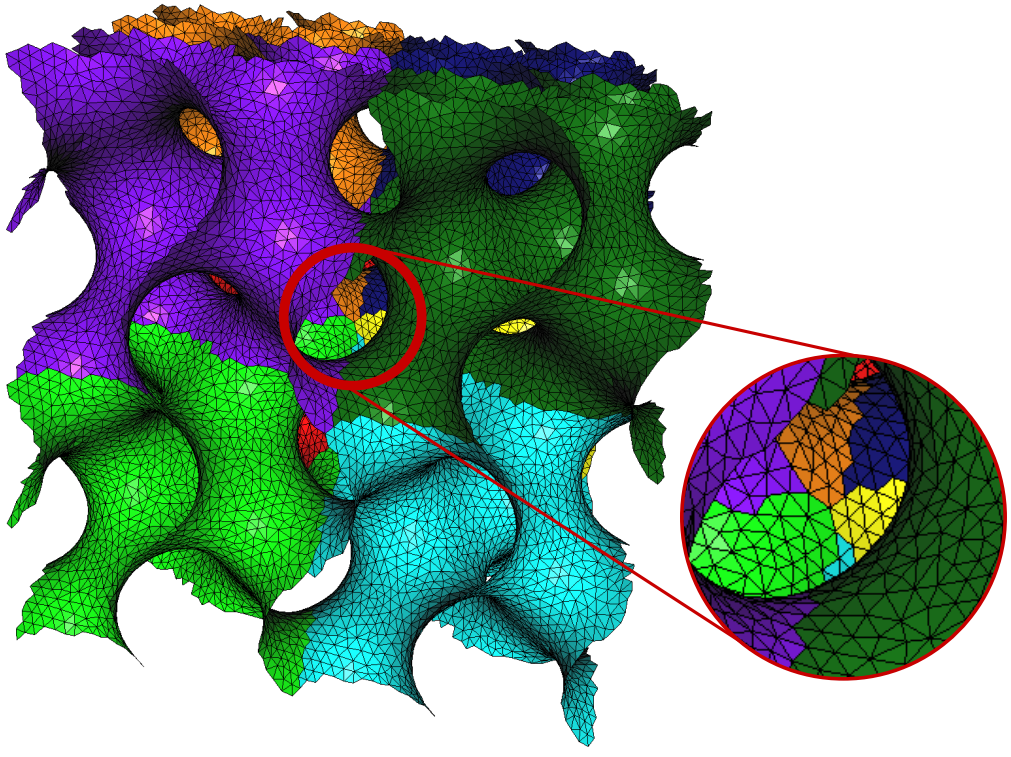
CGAL 3D periodic mesh generation

- code to be polished
- to be submitted to the CGAL editorial board and reviewed
- release expected in CGAL 4.9 (end 2016)

Results

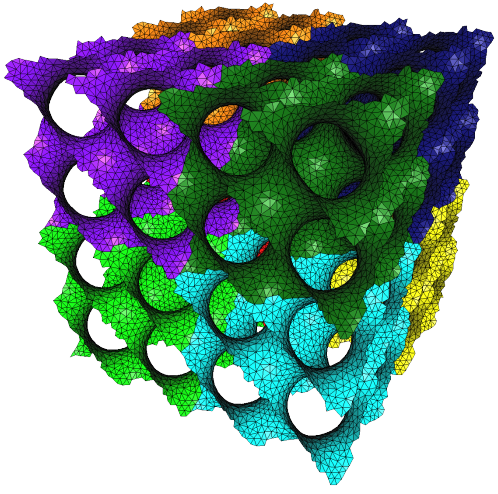


one copy computed

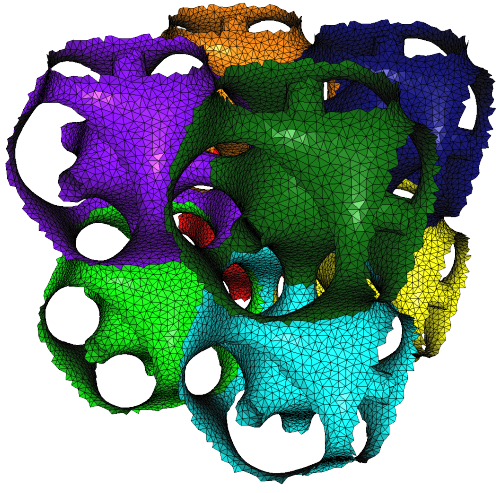


periodic copies fit together

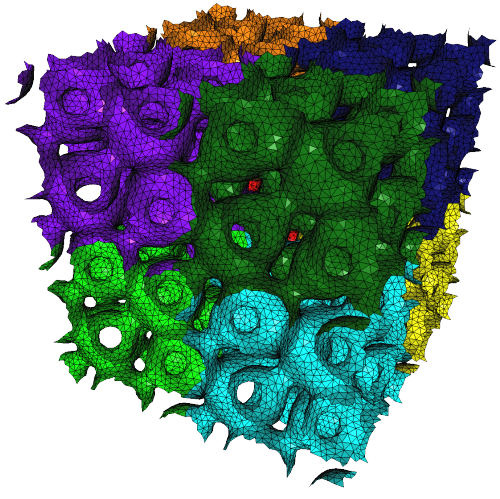
Results



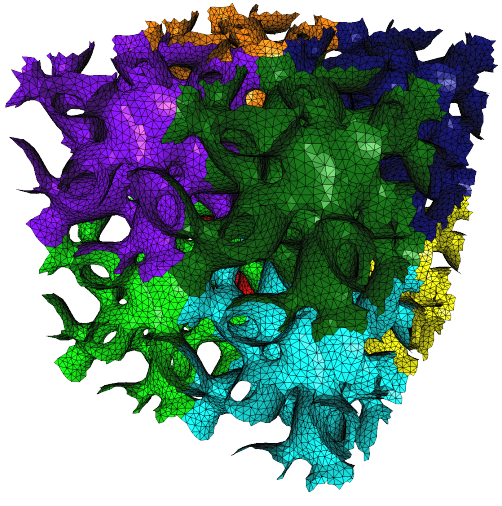
Diamond



Double p



D prime



Lidinoid

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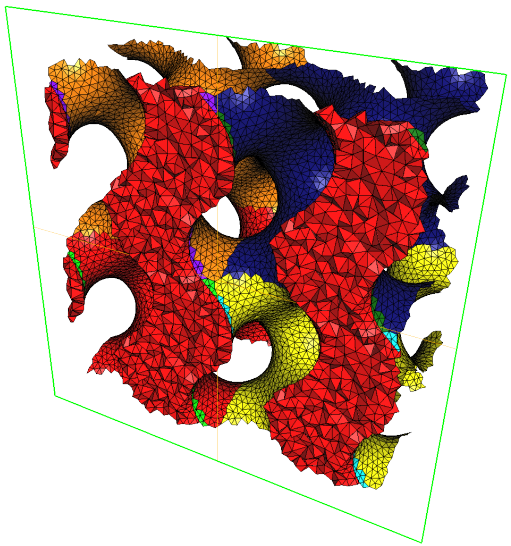
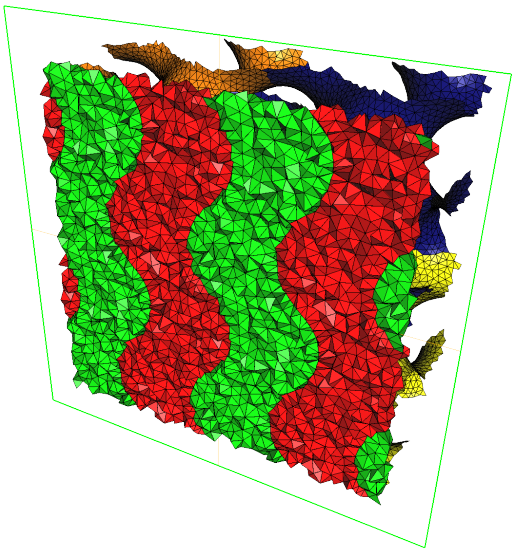
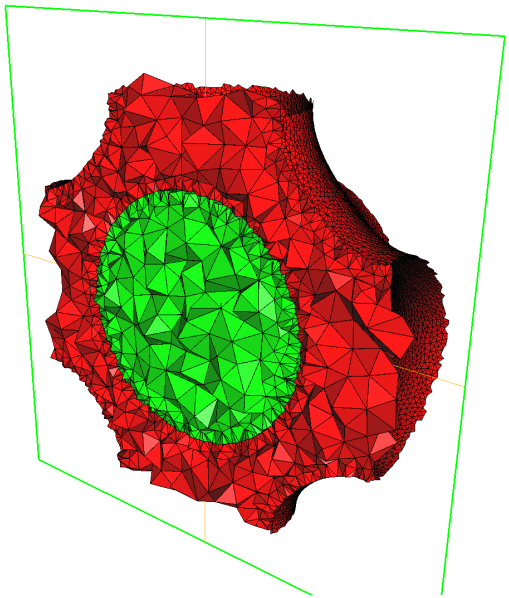


interior

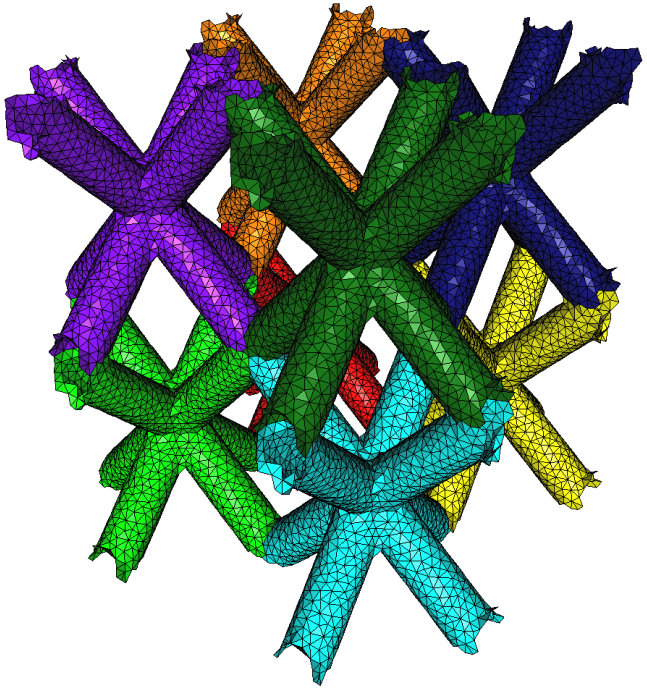


interior and exterior



multi-domain

Results



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data M. Moesen
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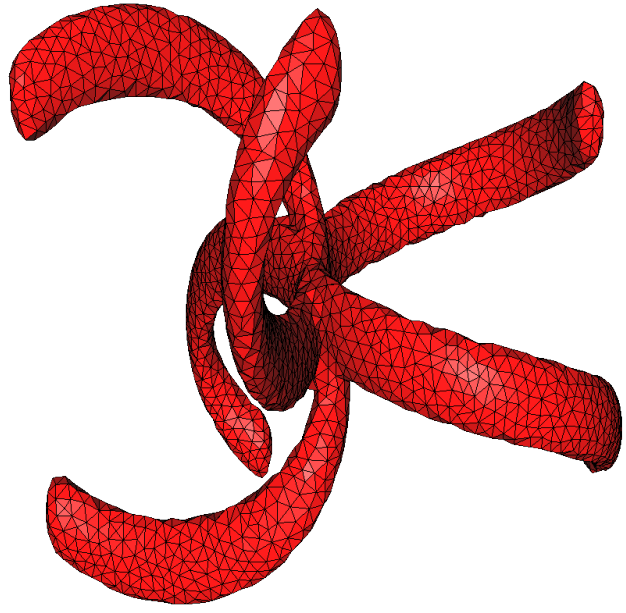

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