Large-scale structure in the Universe and the Power of Voids

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Computational geometry in non-Euclidean spaces, Nancy
Wednesday, August 26, 2015

Part I: The Cosmic Web

- Total perspective vortex
- ~10^{11} stars in our galaxy

Artist impression: Nick Risinger / NASA

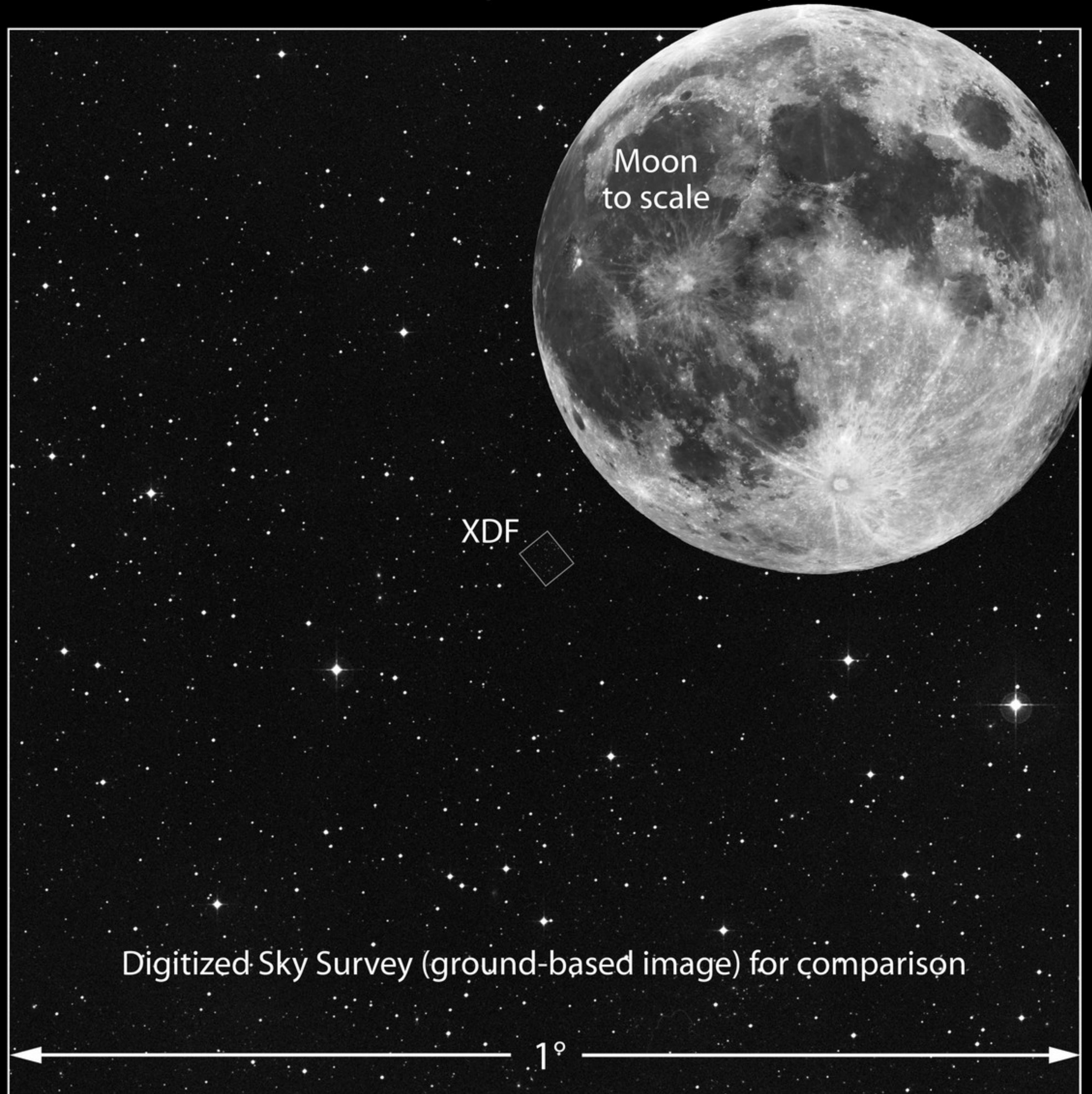
Part I: The Cosmic Web

- Total perspective vortex
- $\sim 10^{11}$ stars in our galaxy
- $\sim 10^{11}$ galaxies in the visible Universe
- Ordinary matter (stars, planets, people) is only 4% of energy budget

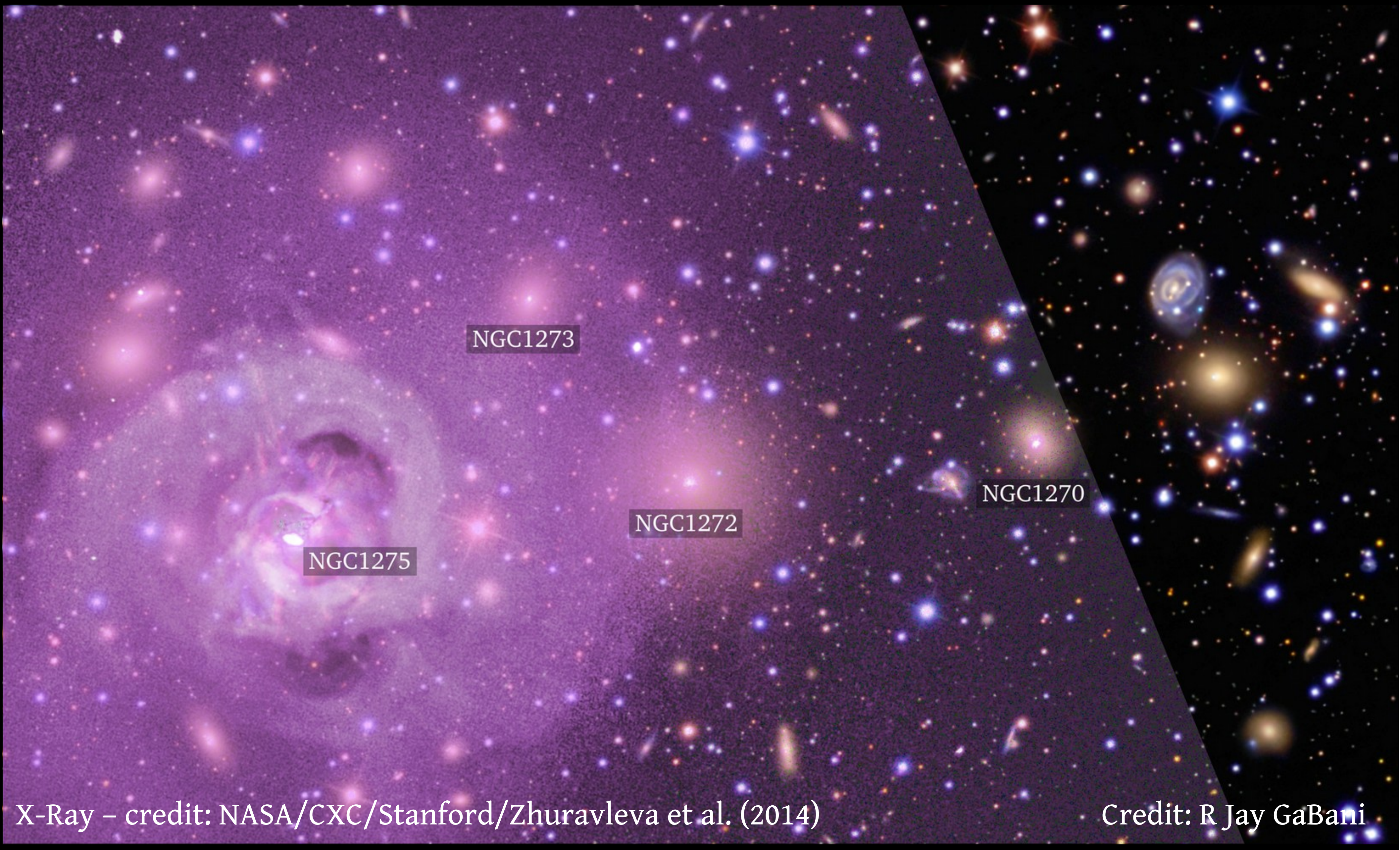
Credit: Lorenzo Comolli


Hubble XDF

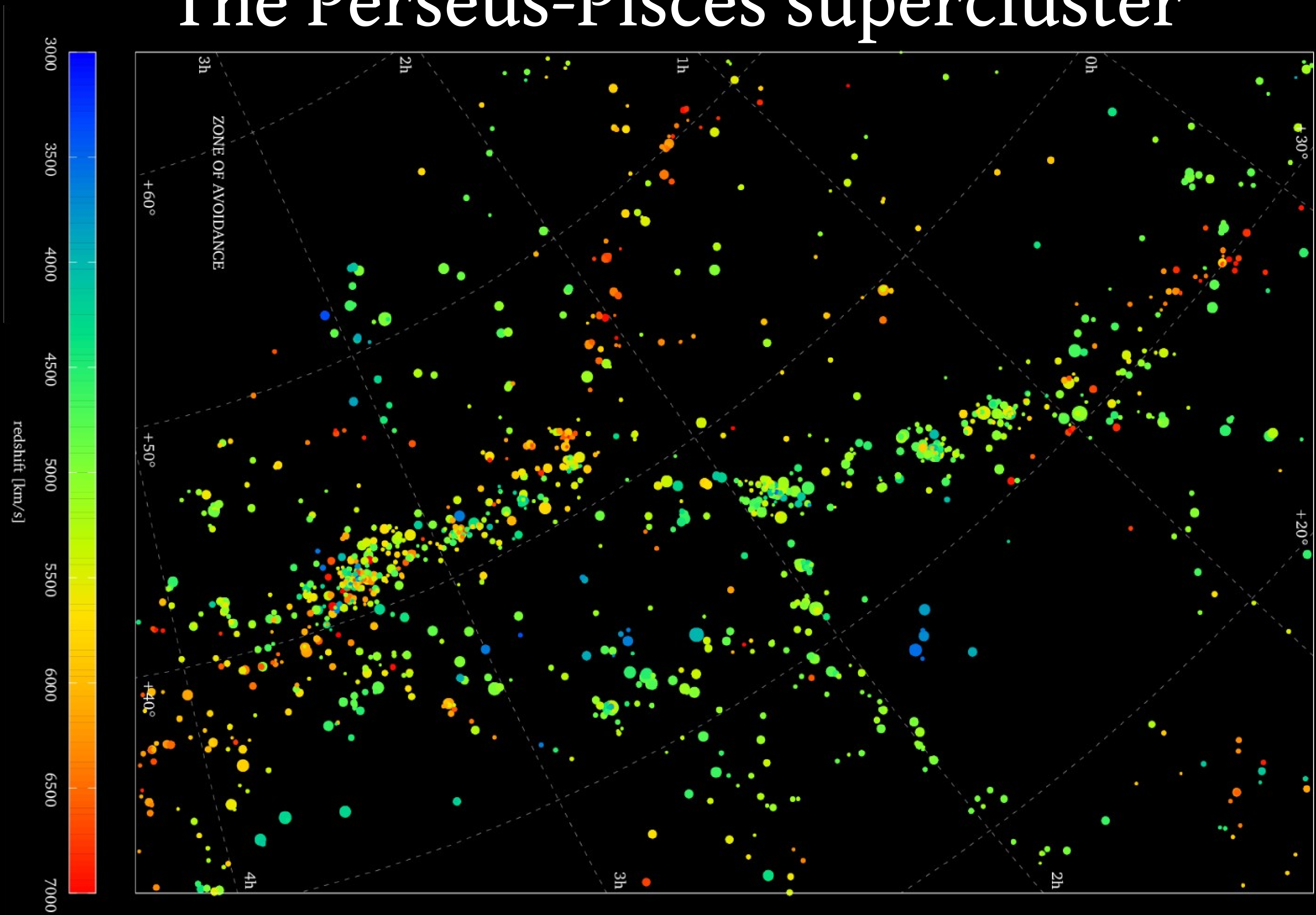
Size of Hubble eXtreme Deep Field on the Sky



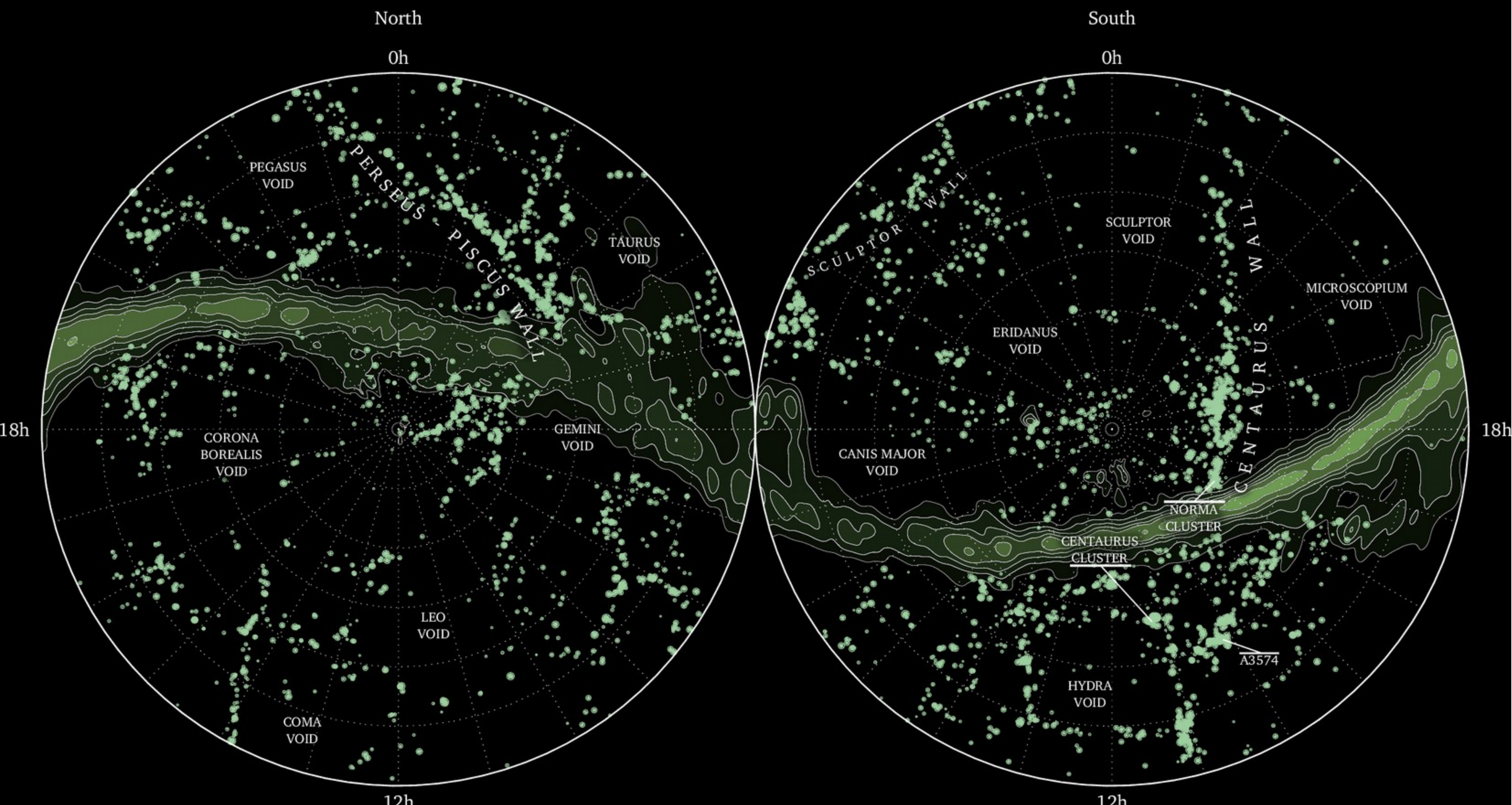
The Perseus cluster



The Perseus-Pisces supercluster

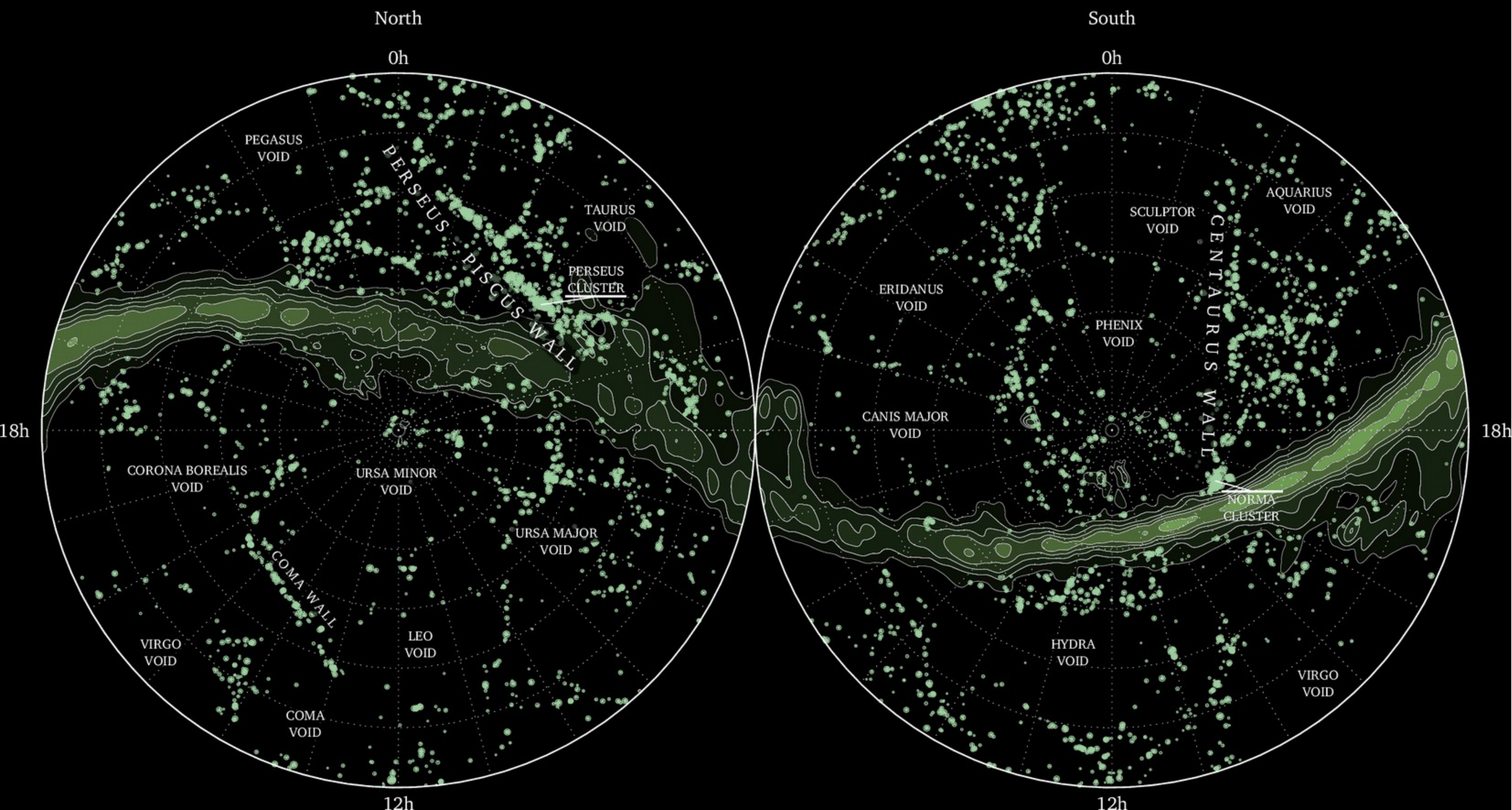


v = 4000-5000 km/s



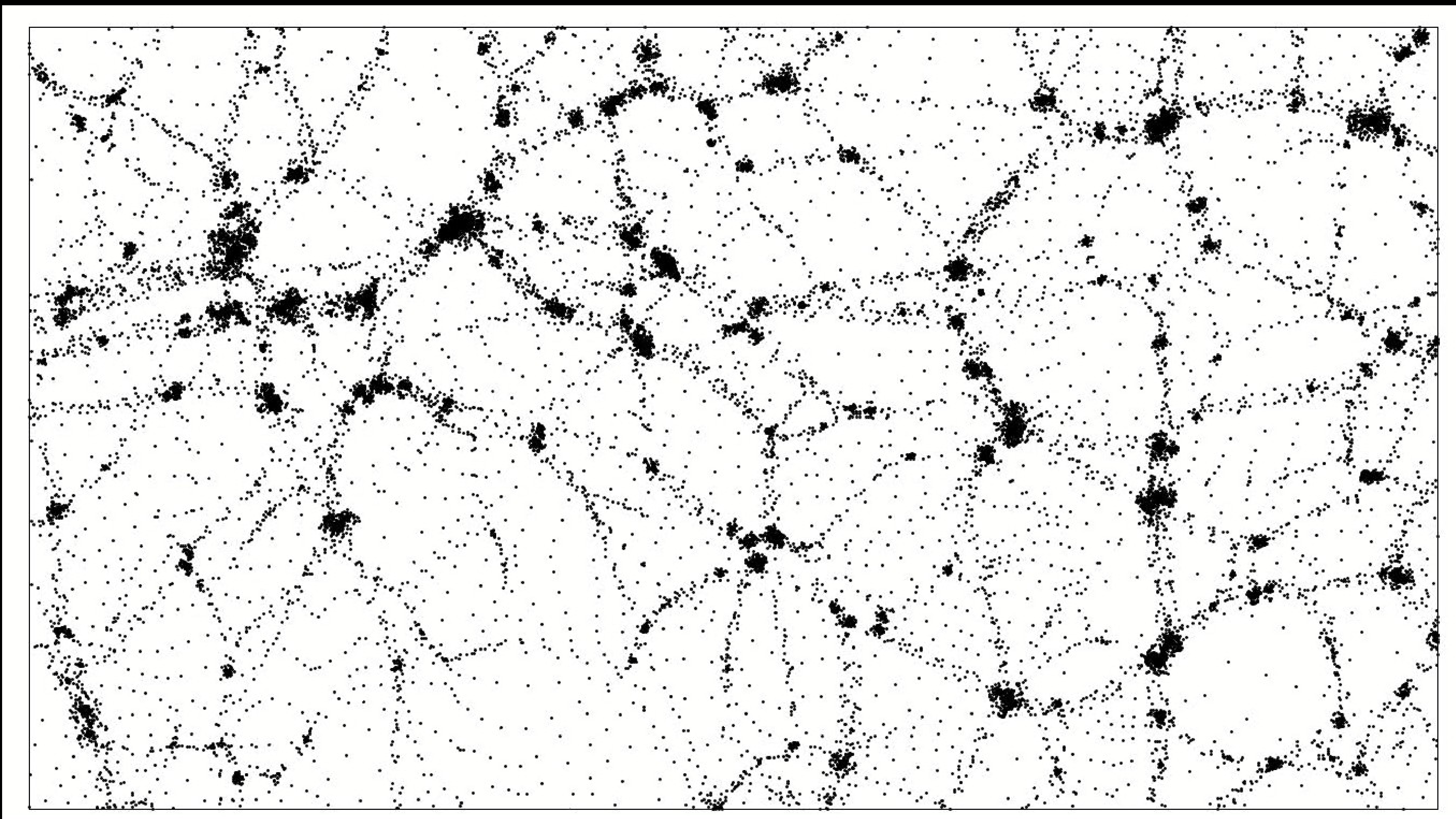
2MRS – Huchra et al. (2012)

v = 5000-6000 km/s

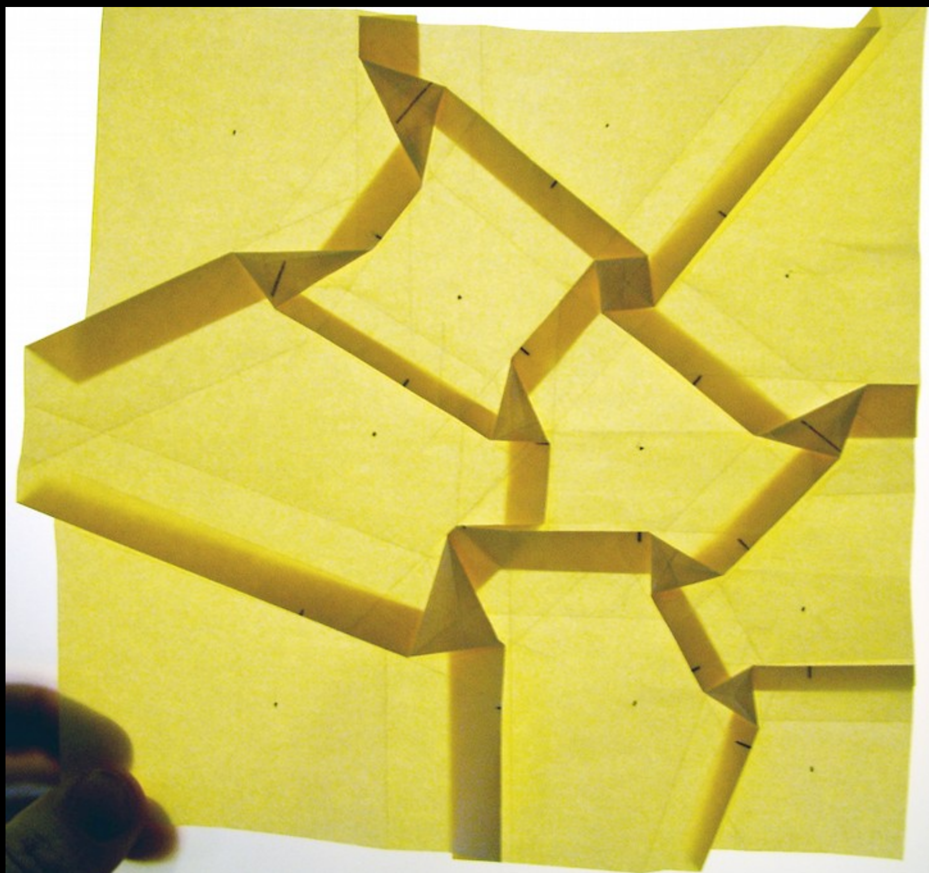


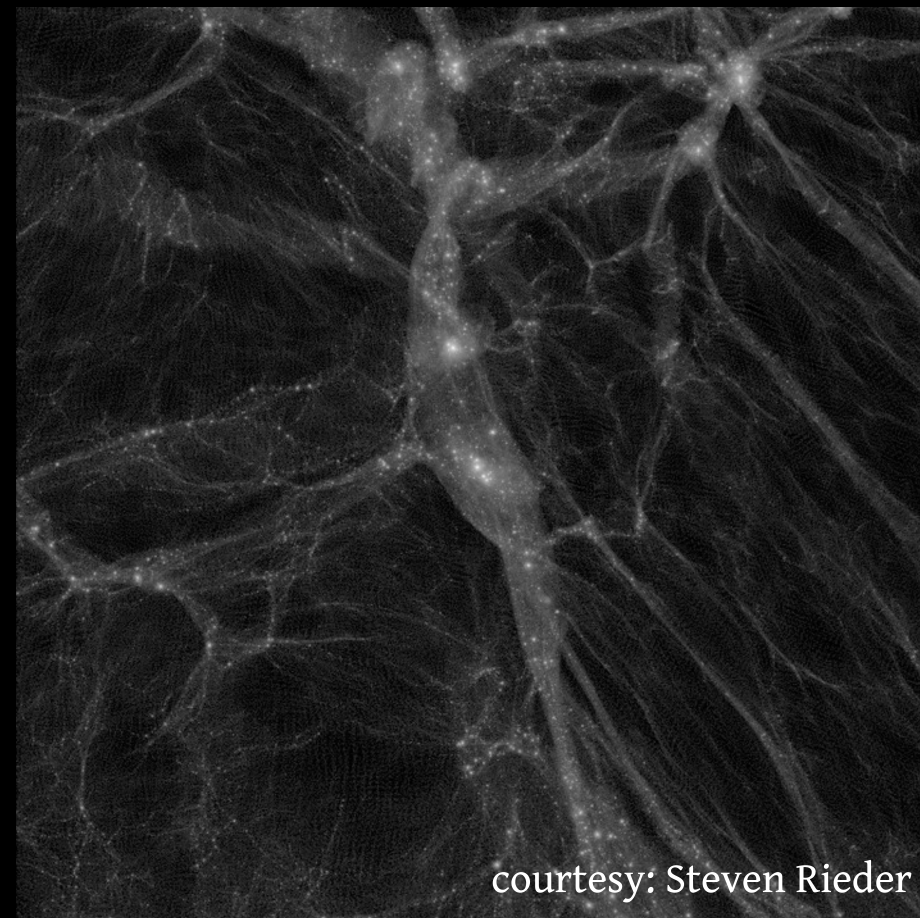
2MRS – Huchra et al. (2012)

Part 2: Structure formation



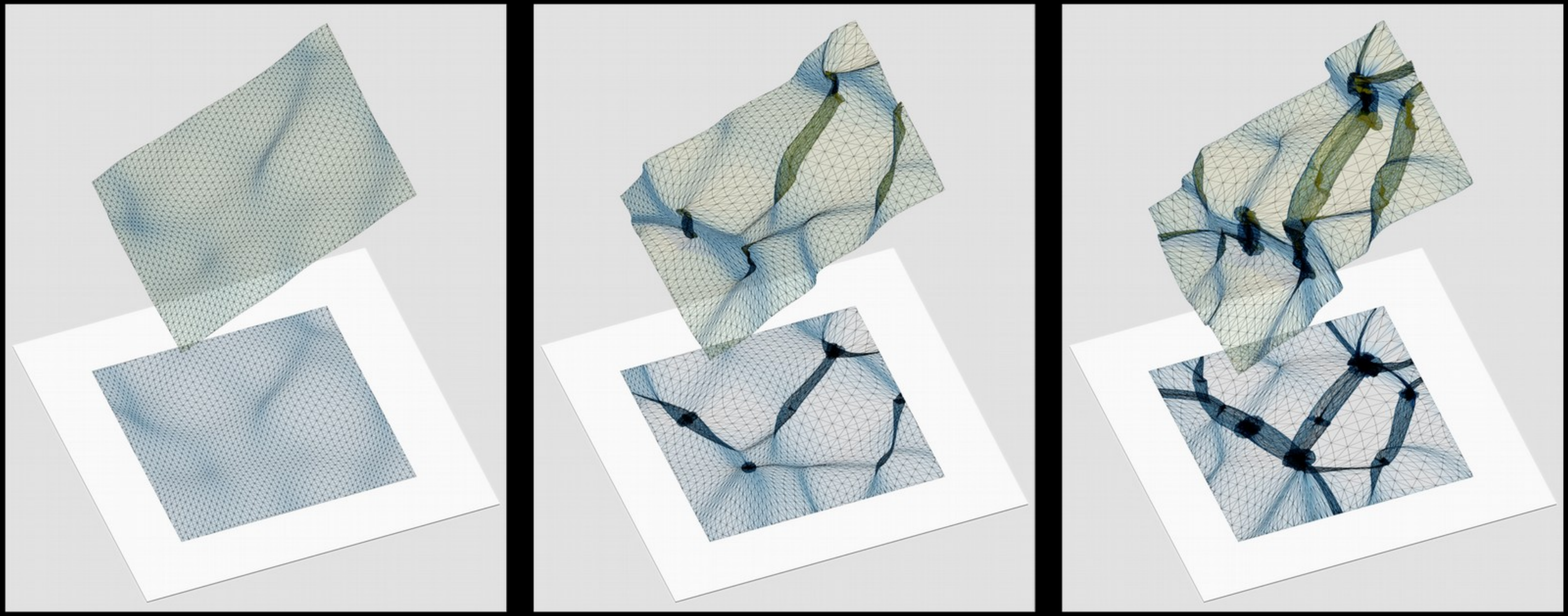
ORIGAMI/Phase-space



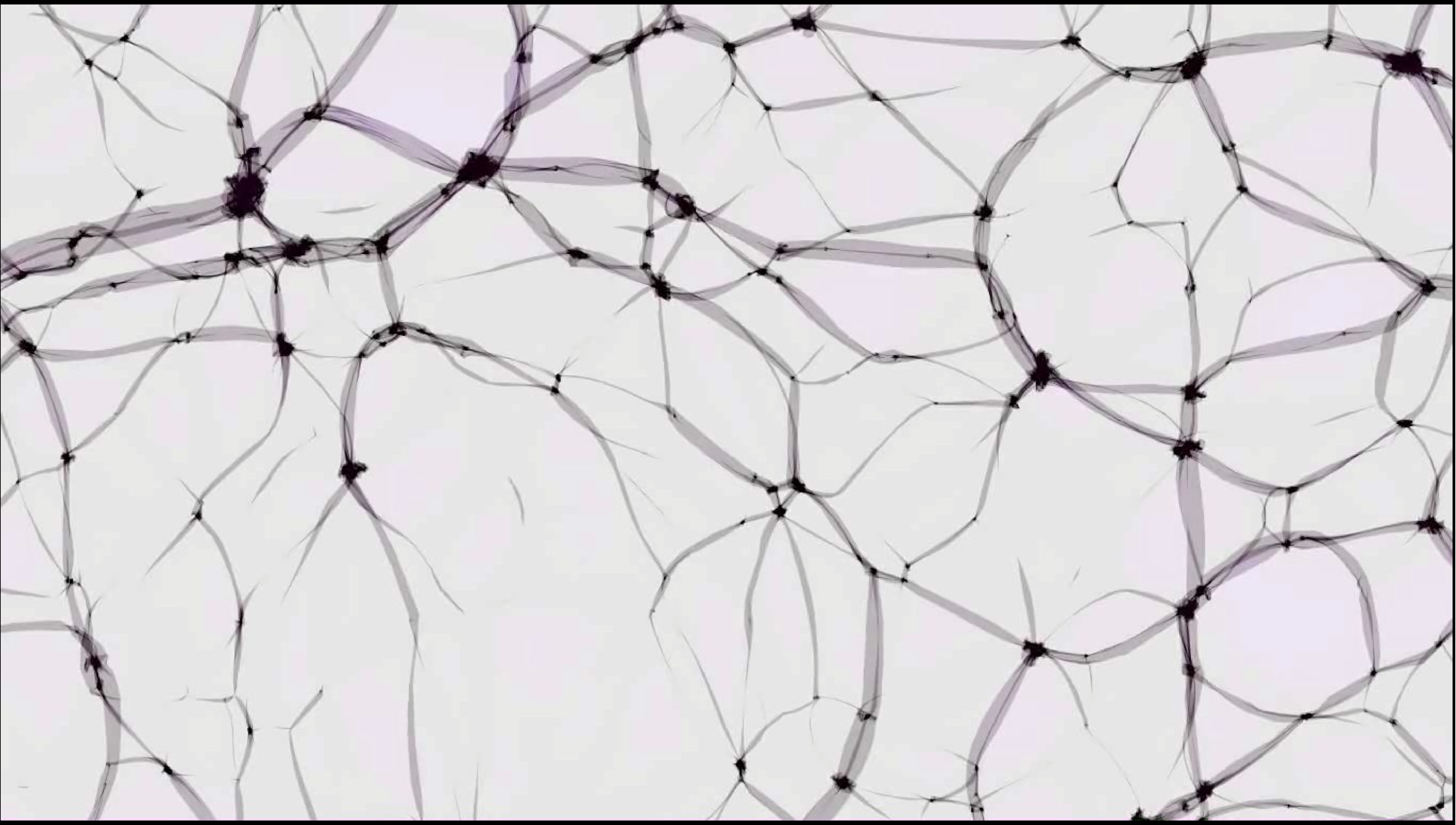


Falck &al. 2012, Neyrinck 2012, Abel &al. 2012, Shandarin &al.2012

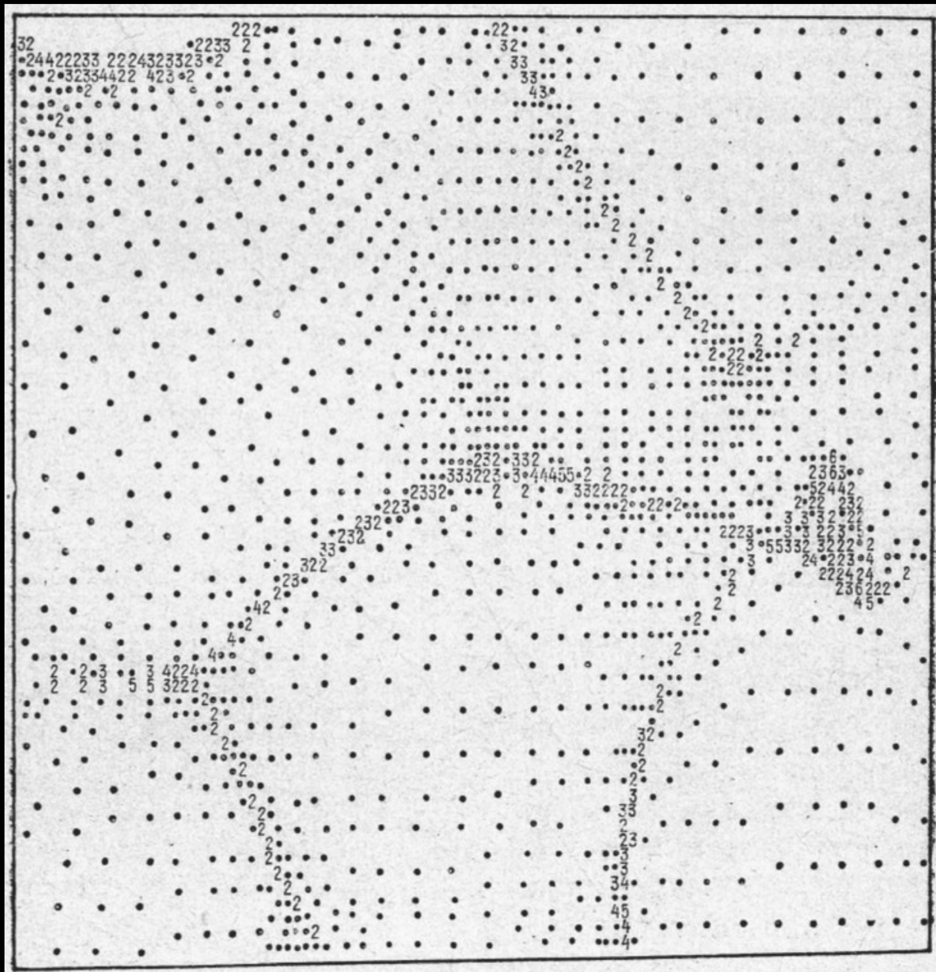
Folding in phase-space

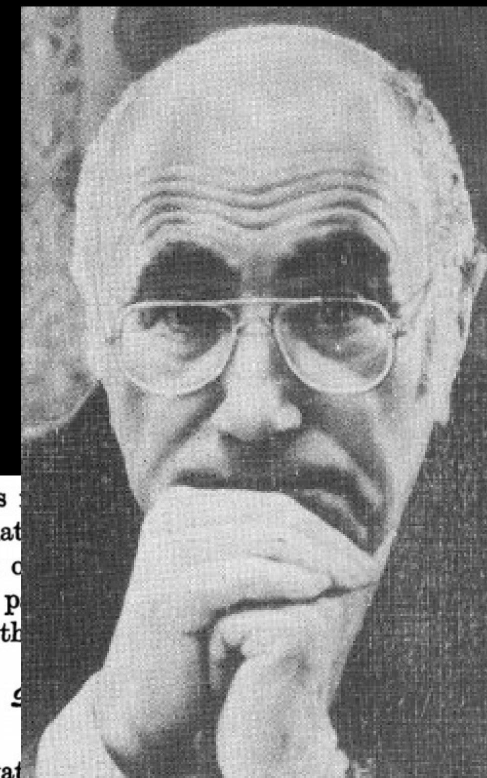


Part II: Structure formation



Zeldovich Approximation





promise to be tedious. Therefore
thod, which gives the right answer
ly, is of interest.
e linear theory is taken to formu-
terms of lagrangian coordinates:
of a particle is given as a function
ordinate $\boldsymbol{q}$ (i.e. its initial position)
$= \boldsymbol{r}(t, \boldsymbol{q})$. The linear theory is
est case of pressure $\mathscr{P} = 0$ ("dust")
in the Newtonian approximation. Only the growing perturbations are considered. The answer is of the form

$$\boldsymbol{r} = a(t)\,\boldsymbol{q} + b(t)\,\boldsymbol{p}(\boldsymbol{q}) . \qquad (1)$$

The first term $a(t)\,\boldsymbol{q}$ describes the cosmological
term describes the perturba-
t) and $b(t)$ are known; $b(t)$ is
han $a(t)$, as a result of gravita-
vector function $\boldsymbol{p}(\boldsymbol{q})$ depends
$\frac{\partial p_i}{\partial q_k} = \frac{\partial p_k}{\partial q_i}$ in the growing mode
$= \xi_1$, $\beta = \xi_2$, and $\gamma = \xi_3$ are the
$| = 0$. The sign of α, β, γ is not
ner, for the sake of subsequent

Let us … he
approximat … si-
bility. In c … all
group of p … ve
calculate th …

The derivat … es.
After choosing the coordinate system along the axes, one obtains[1]) for a given $\boldsymbol{q}$

$$D = \left\| \begin{matrix} a(t) - \alpha\, b(t) & 0 & 0 \\ 0 & a(t) - \beta\, b(t) & 0 \\ 0 & 0 & a(t) - \gamma\, b(t) \end{matrix} \right\| .$$

A volume which was initially a cube (at $t \to 0$) and which would be a cube in the unperturbed motion, is transformed into a parallelepiped. One can always choose the axis of the cube so that it is transformed into a rectangular parallelepiped; the axes are not rotating in solution (1). The density near a particle with given $\boldsymbol{q}$ is given by the conservation of mass

$$\varrho (a - \alpha b)\,(a - \beta b)\,(a - \gamma b) = \bar{\varrho} a^3 . \qquad (2)$$

- Zeldovich 1970
- Doroshkevich, Sunyaev & Zeldovich 1976
- Zeldovich & Shandarin 1989

Zeldovich Approximation

$$\boldsymbol{x} = \boldsymbol{q} - t\boldsymbol{\nabla}_q \Phi(\boldsymbol{q})$$

Current particle position x(t) → *Eulerian space*

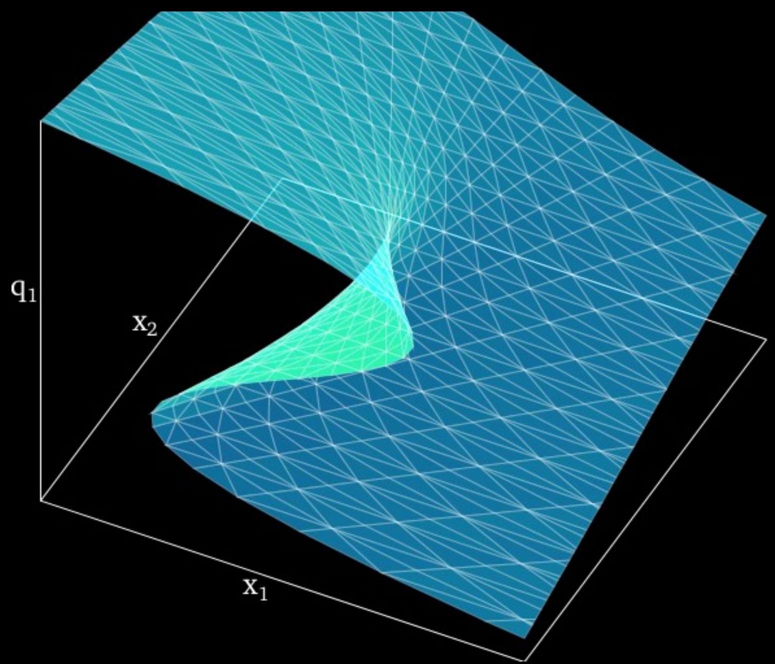
Starting position: q = x(0) → *Lagrangian space*

Constant particle velocity $\boldsymbol{v} = -\boldsymbol{\nabla}\Phi$

Equation of motion: $$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}_x)\,\boldsymbol{v} = 0$$

$$\boldsymbol{x} = \boldsymbol{\nabla}_q \left(\frac{q^2}{2} - t\Phi(q) \right) = \boldsymbol{\nabla}_q \varphi(q, t)$$

Caustic formation → Pancakes!



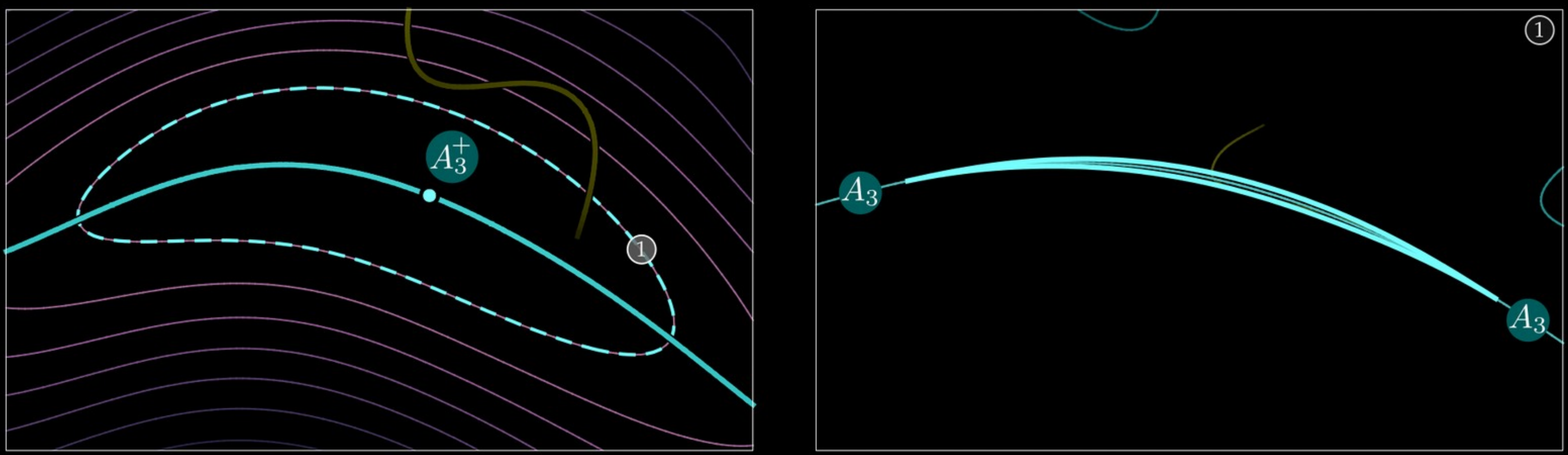
$$\mathcal{L} : \boldsymbol{q} \mapsto \boldsymbol{x}$$

$$\boldsymbol{x}(\boldsymbol{q}, t) = \boldsymbol{q} - t\boldsymbol{\nabla}_q \Phi(\boldsymbol{q})$$

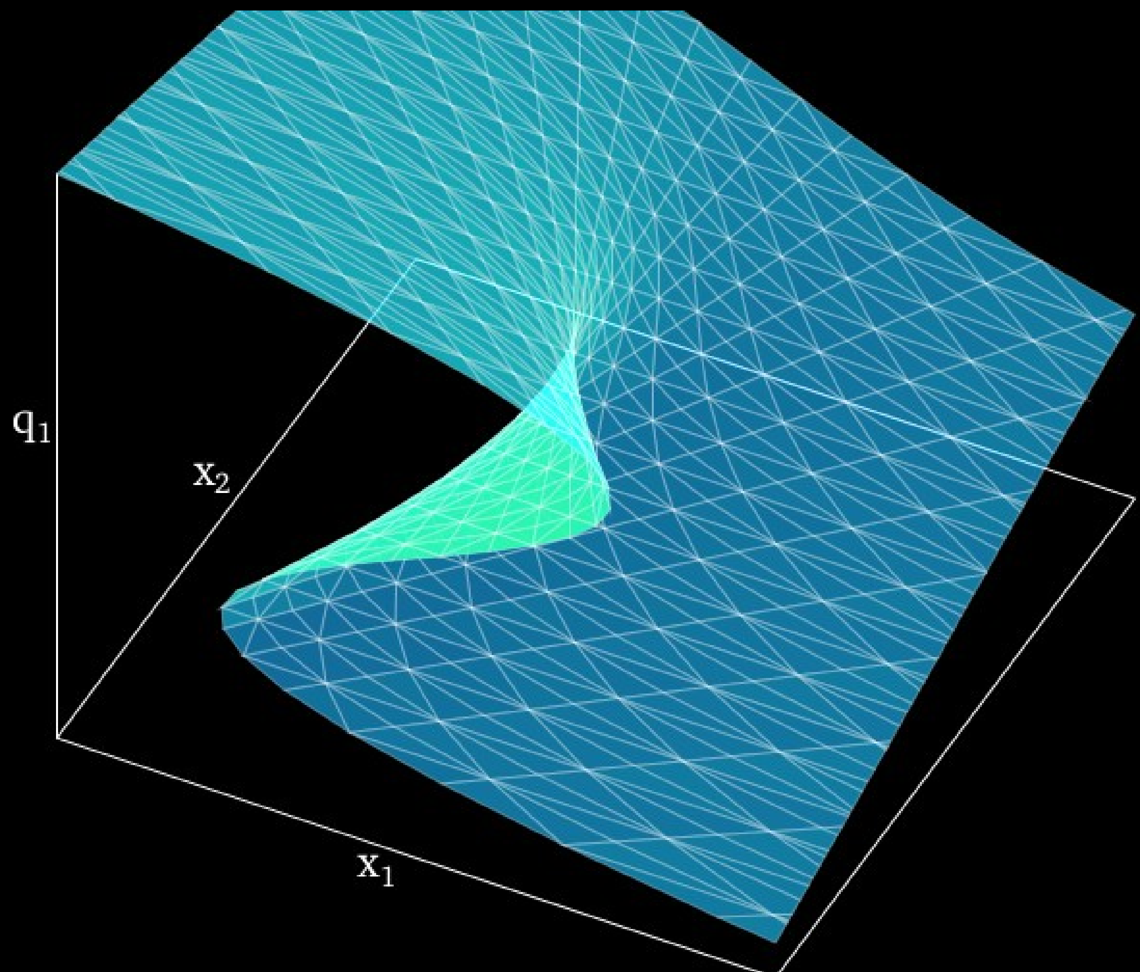
$$\rho(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = \langle\rho\rangle \mathrm{d}\boldsymbol{q}$$

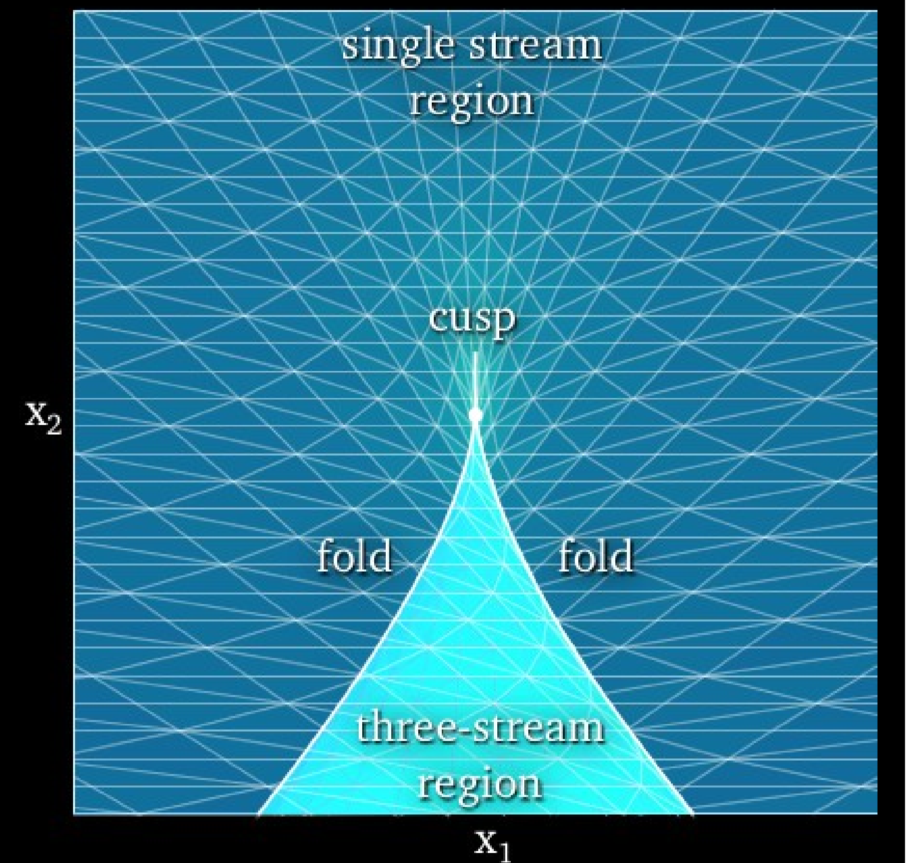
$$\begin{aligned}
\frac{\rho(\boldsymbol{x})}{\langle\rho\rangle} &= \sum_{q^\star \in \mathcal{L}^{-1}(x)} \left| \frac{\partial x_i}{\partial q_j} \right|^{-1}_{q=q^\star} \\
&= \sum_{q^\star \in \mathcal{L}^{-1}(x)} \left| \begin{matrix} 1 - t\alpha(\boldsymbol{q}) & 0 & 0 \\ 0 & 1 - t\beta(\boldsymbol{q}) & 0 \\ 0 & 0 & 1 - t\gamma(\boldsymbol{q}) \end{matrix} \right|^{-1}_{q=q^\star}
\end{aligned}$$

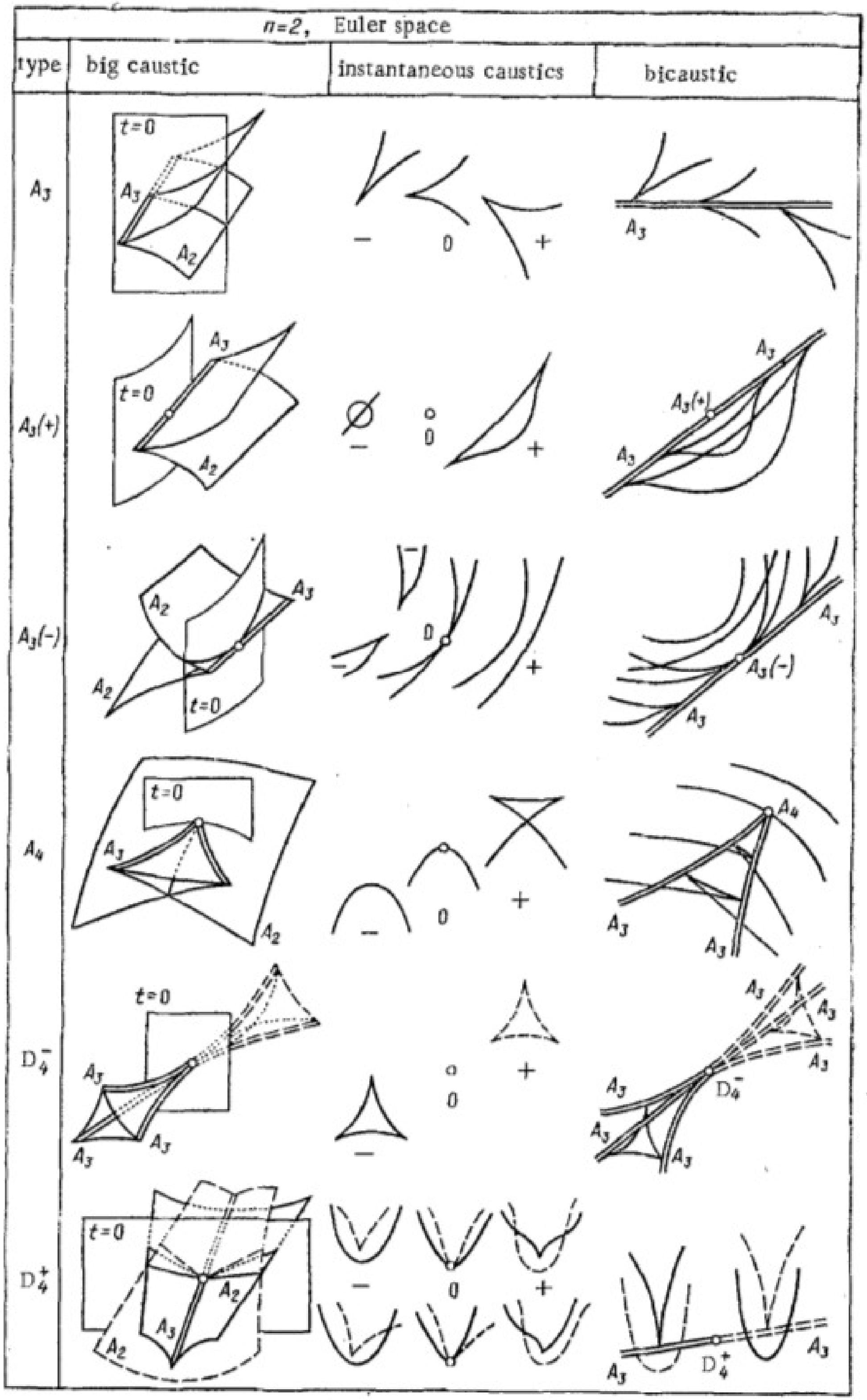
Pancakes!



Mathematics of folds in phase-space

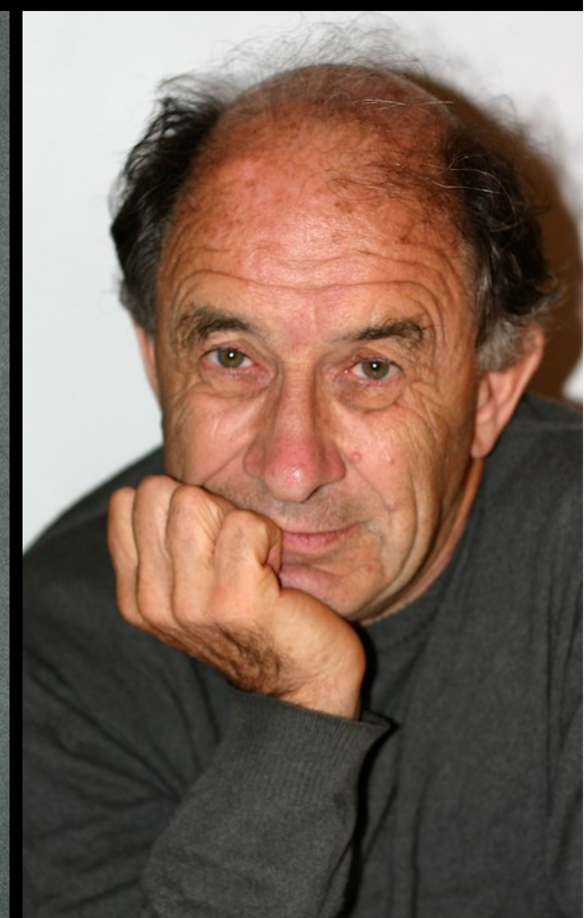






Zeldovich Approximation

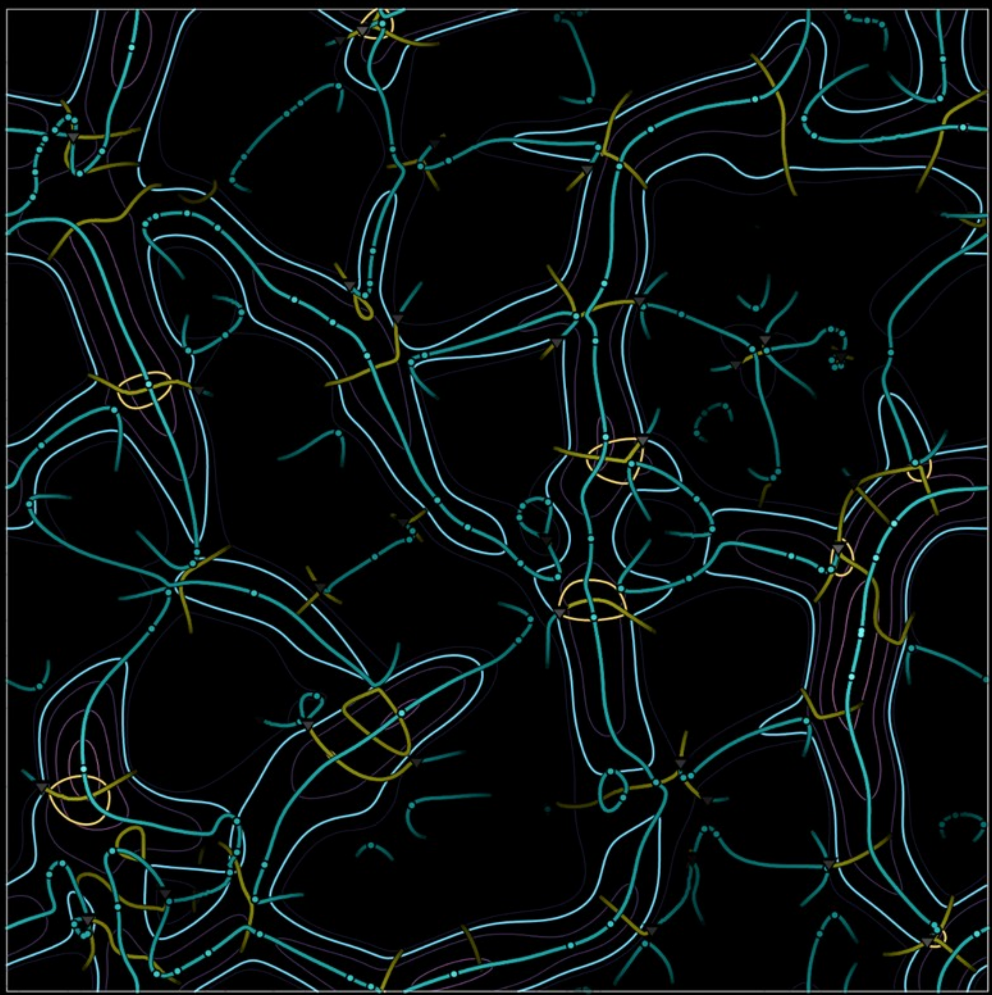
- René Thom 1972 → Catastrophe Theory
- Arnold 1976 → Lagrangian Catastrophes
- Arnold, Zeldovich & Shandarin 1982
- Arnold 1986, 1992

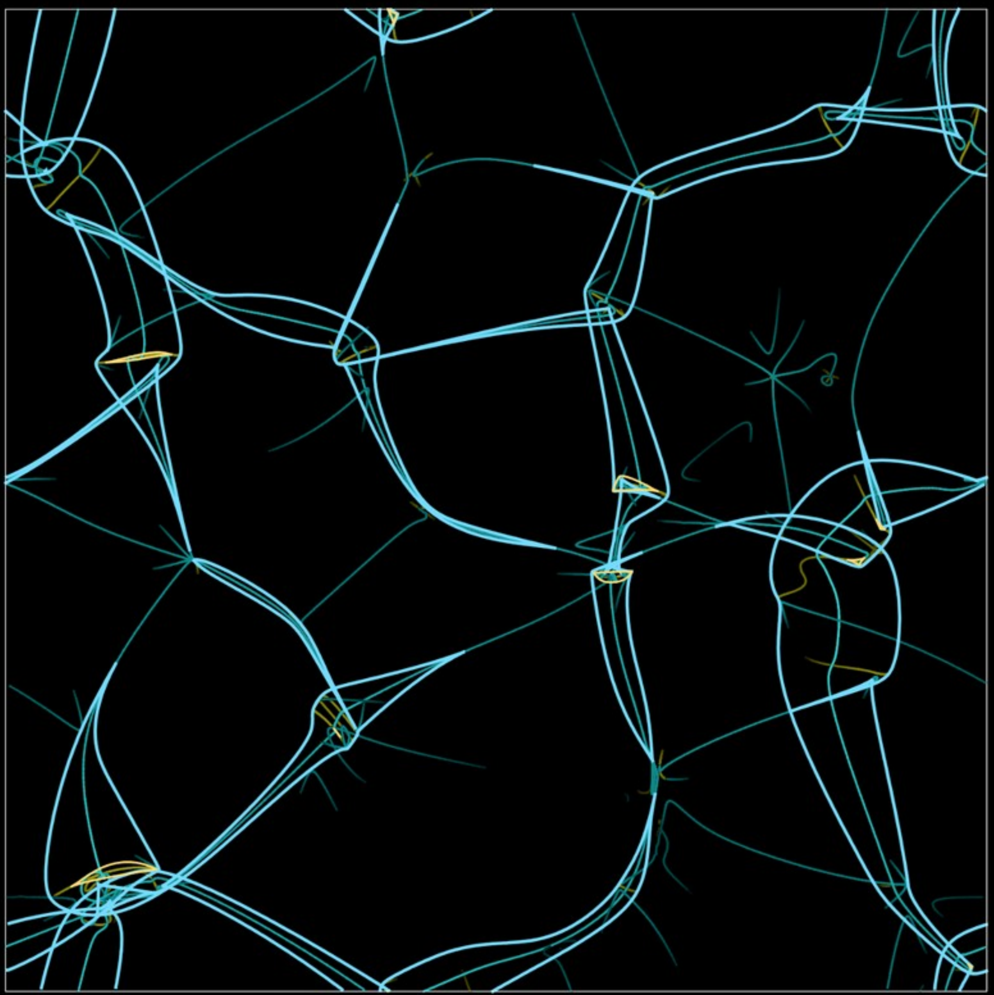


A_3-lines

Lagrangian space

Eulerian space

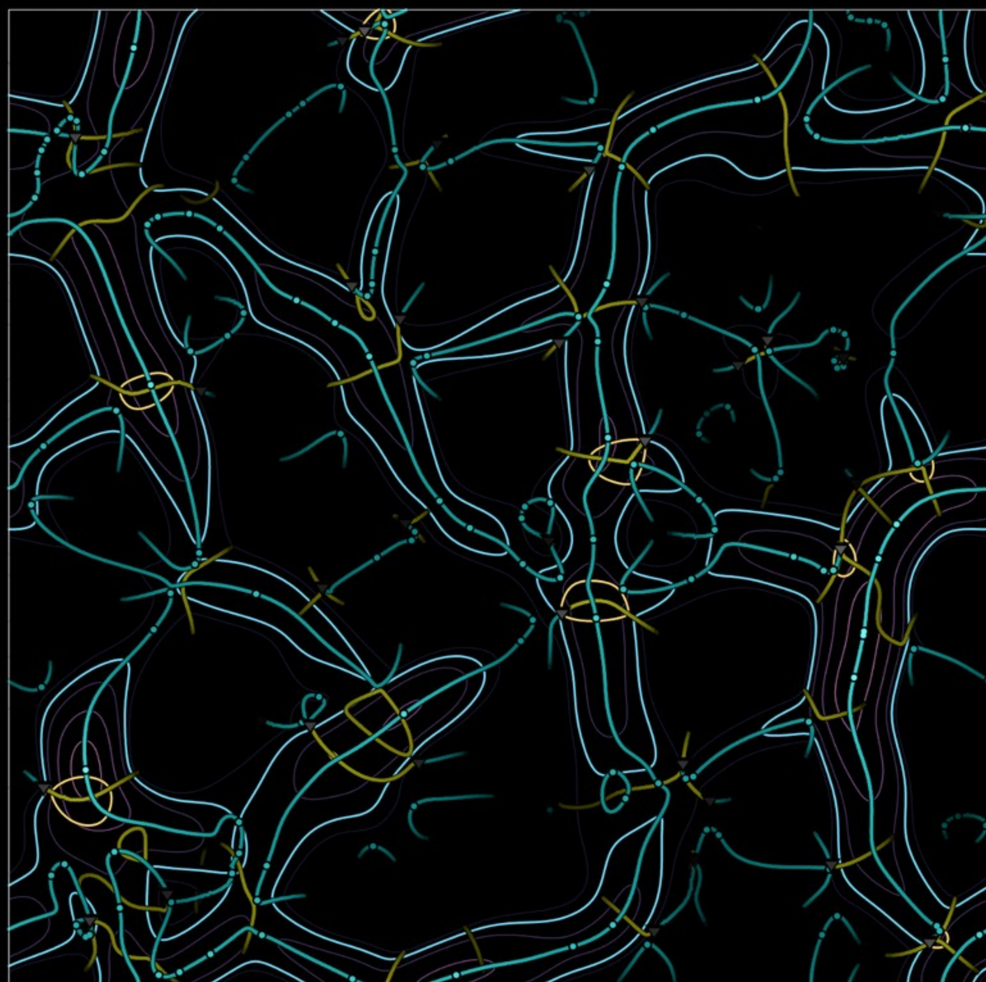




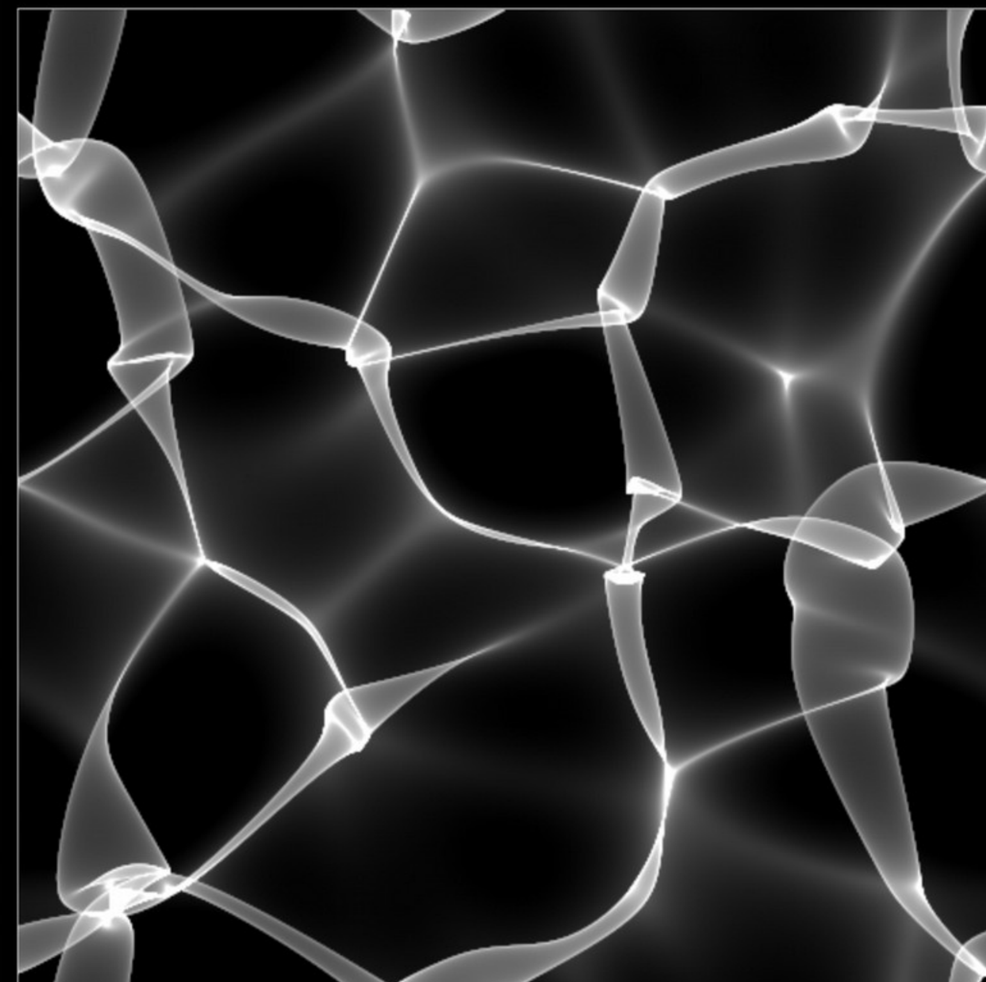
Hidding, Shandarin & van de Weygaert 2014

A_3-lines

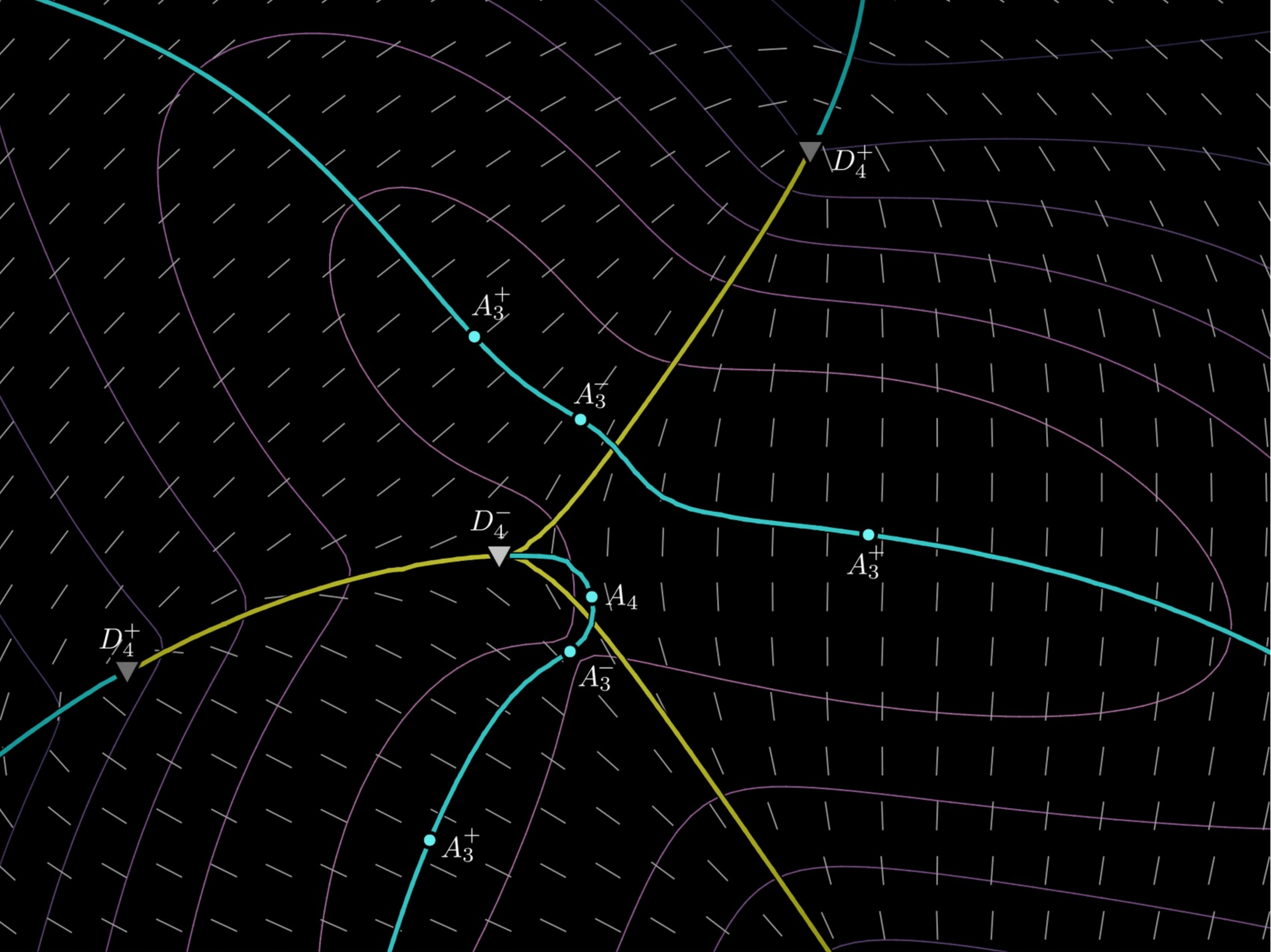
Lagrangian space



Eulerian space

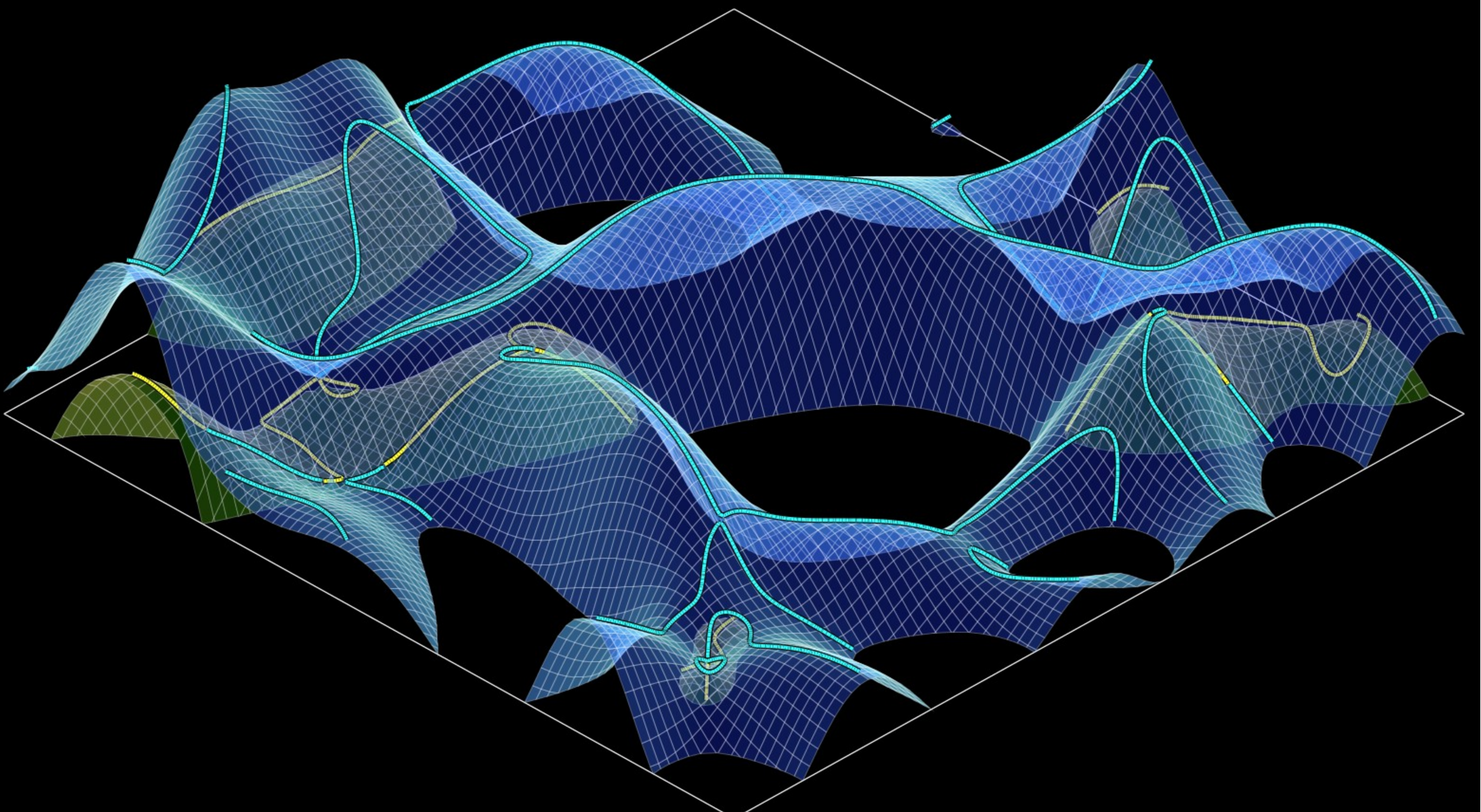


Hidding, Shandarin & van de Weygaert 2014

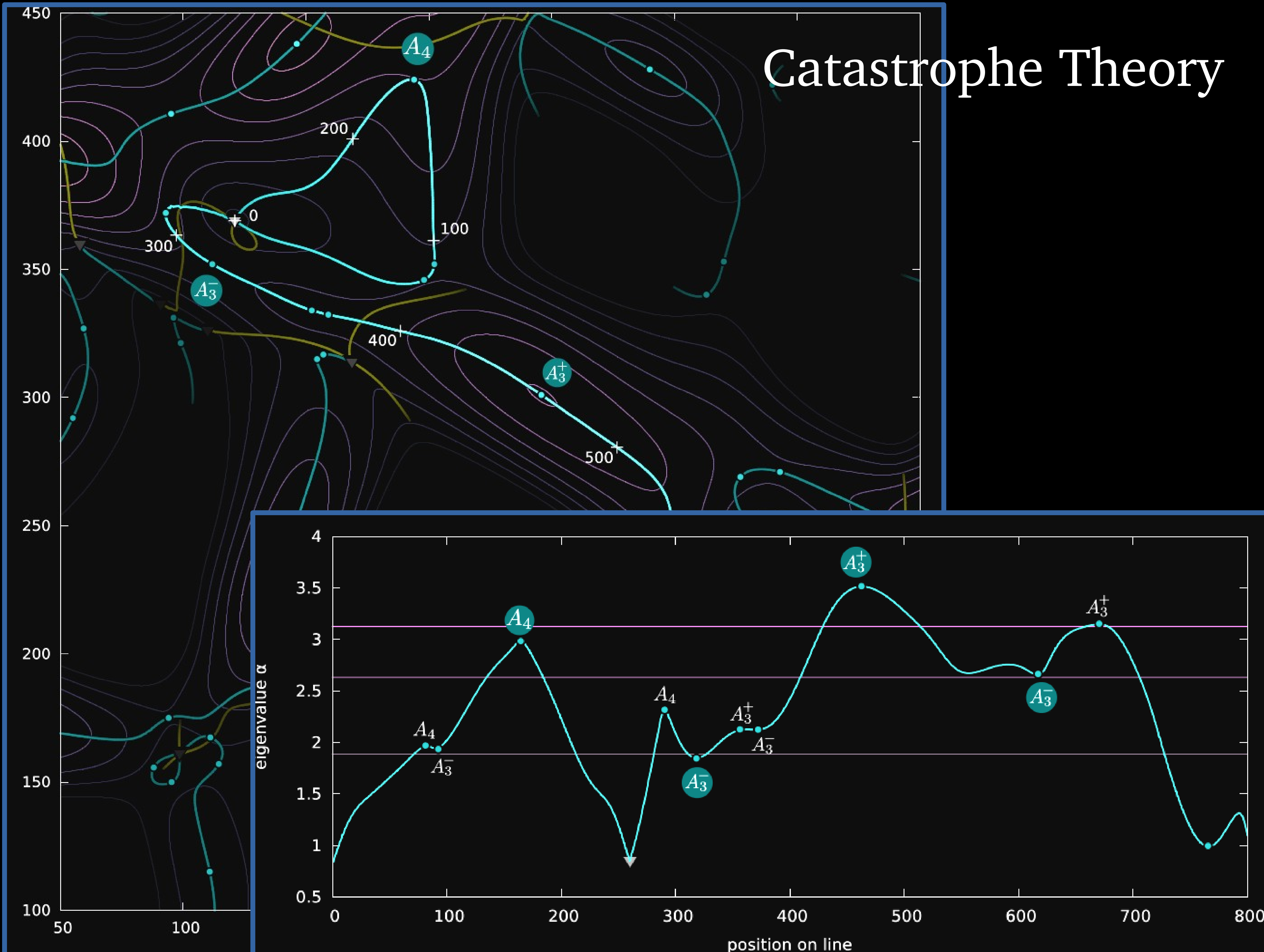


D_4^+
A_3^+
A_3^-
D_4^-
A_3^+
A_4
D_4^+
A_3^-
A_3^+

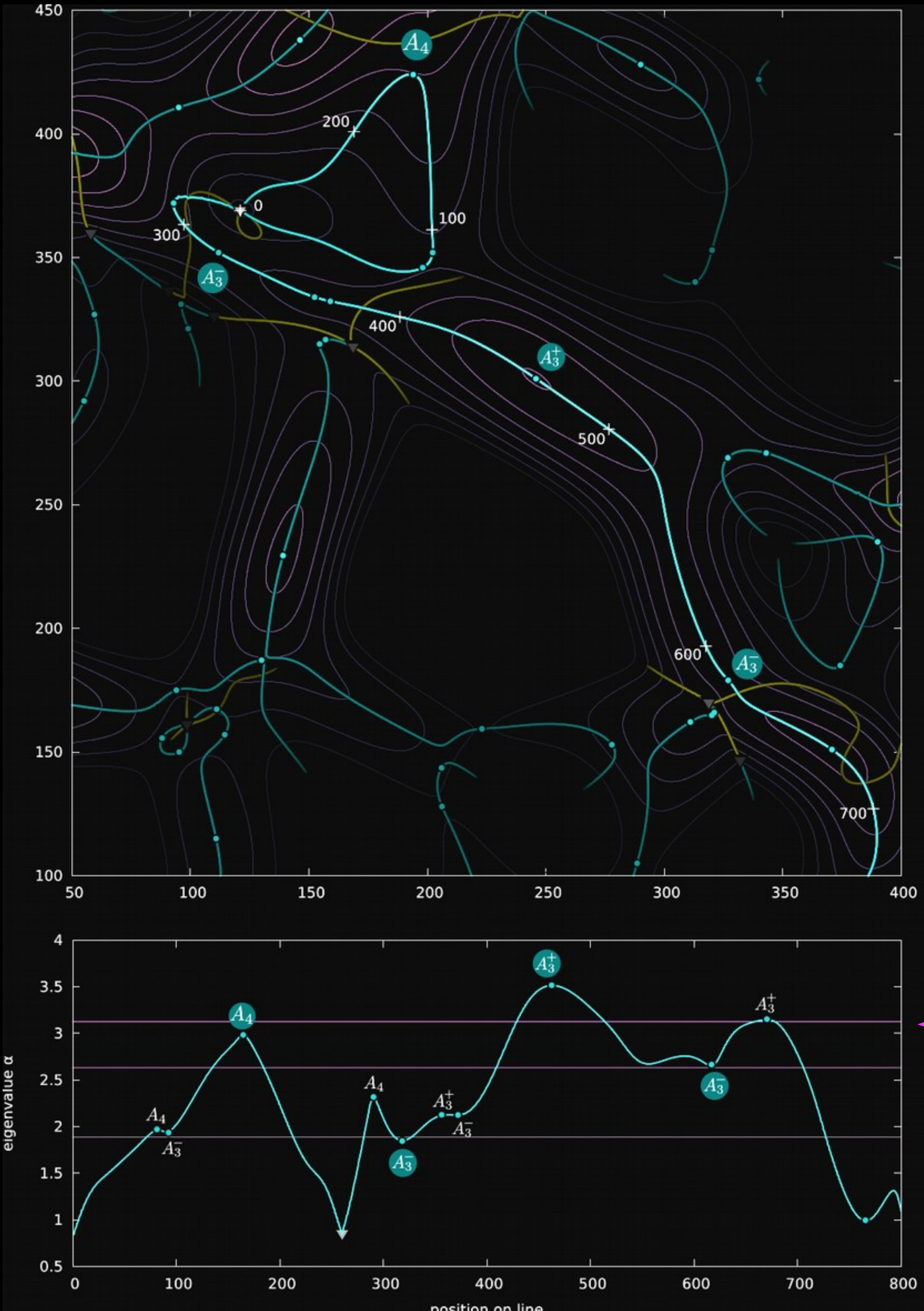
Eigenvalue landscape

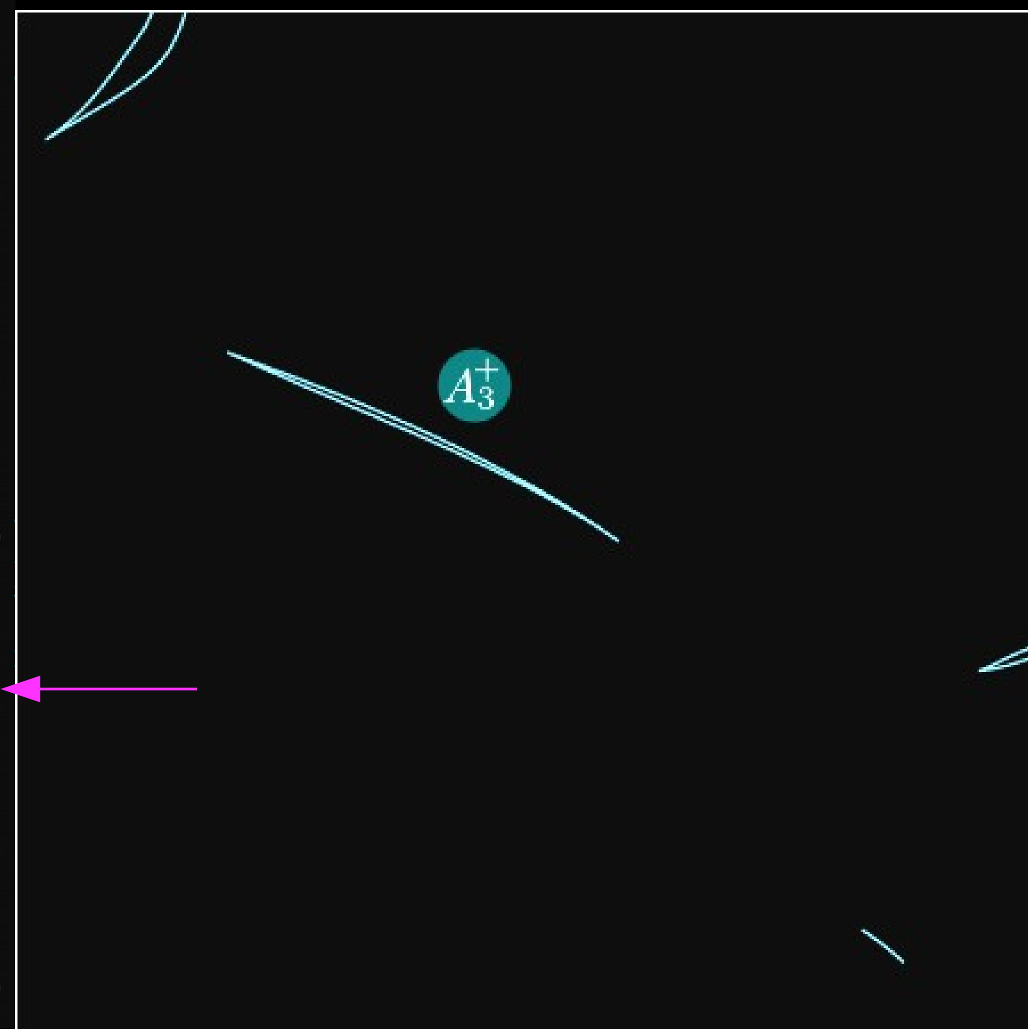


Catastrophe Theory

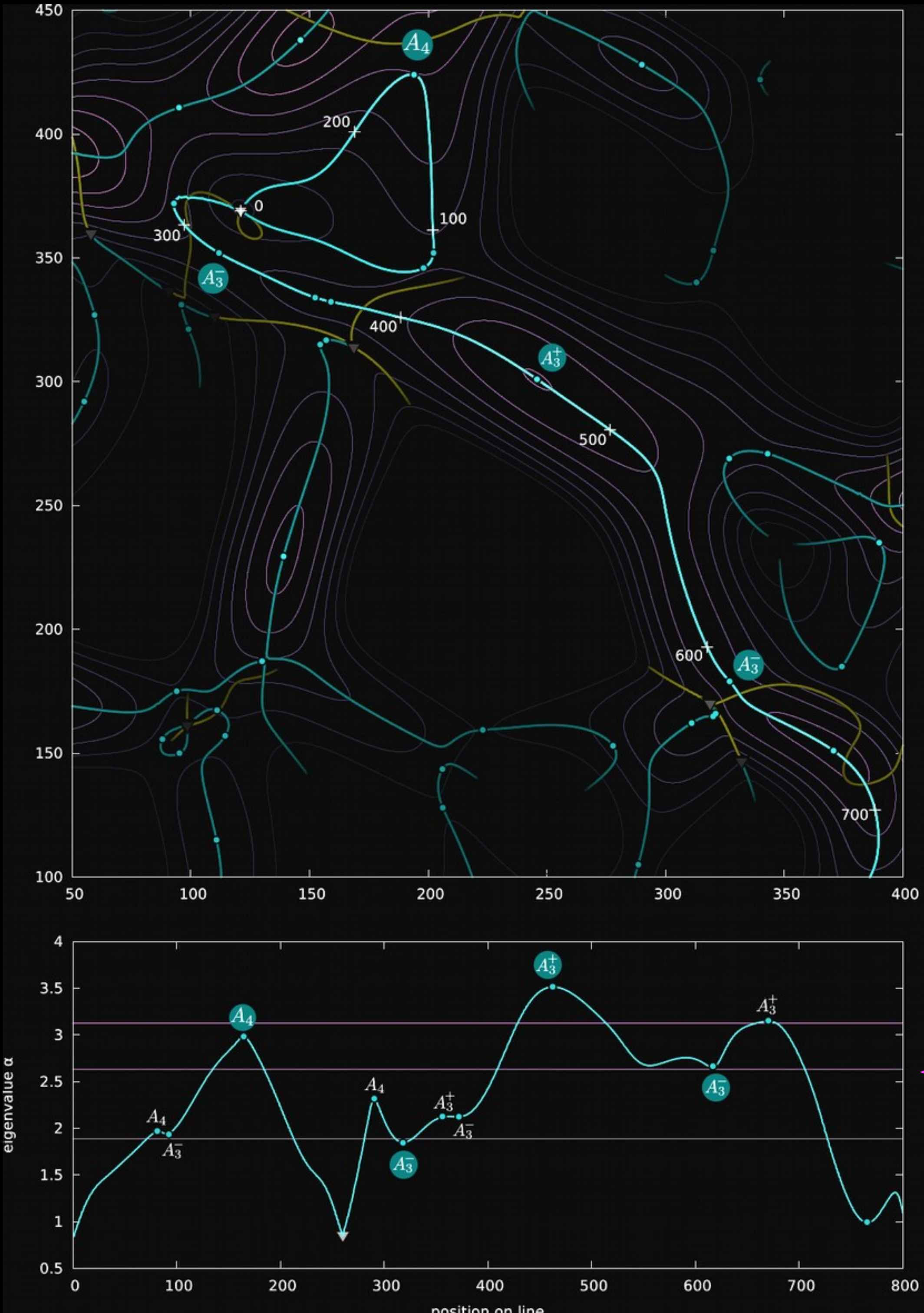


Catastrophe Theory

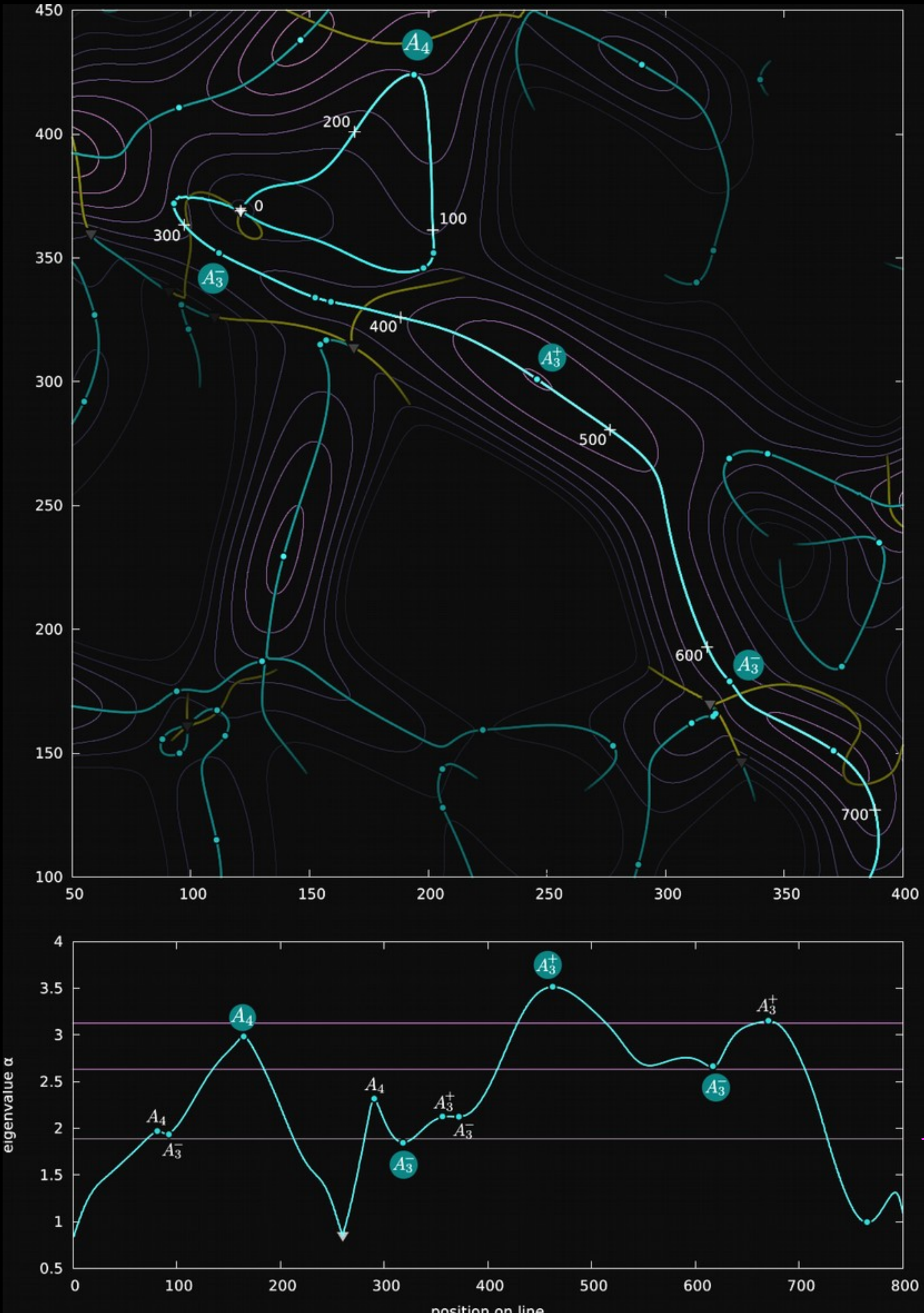




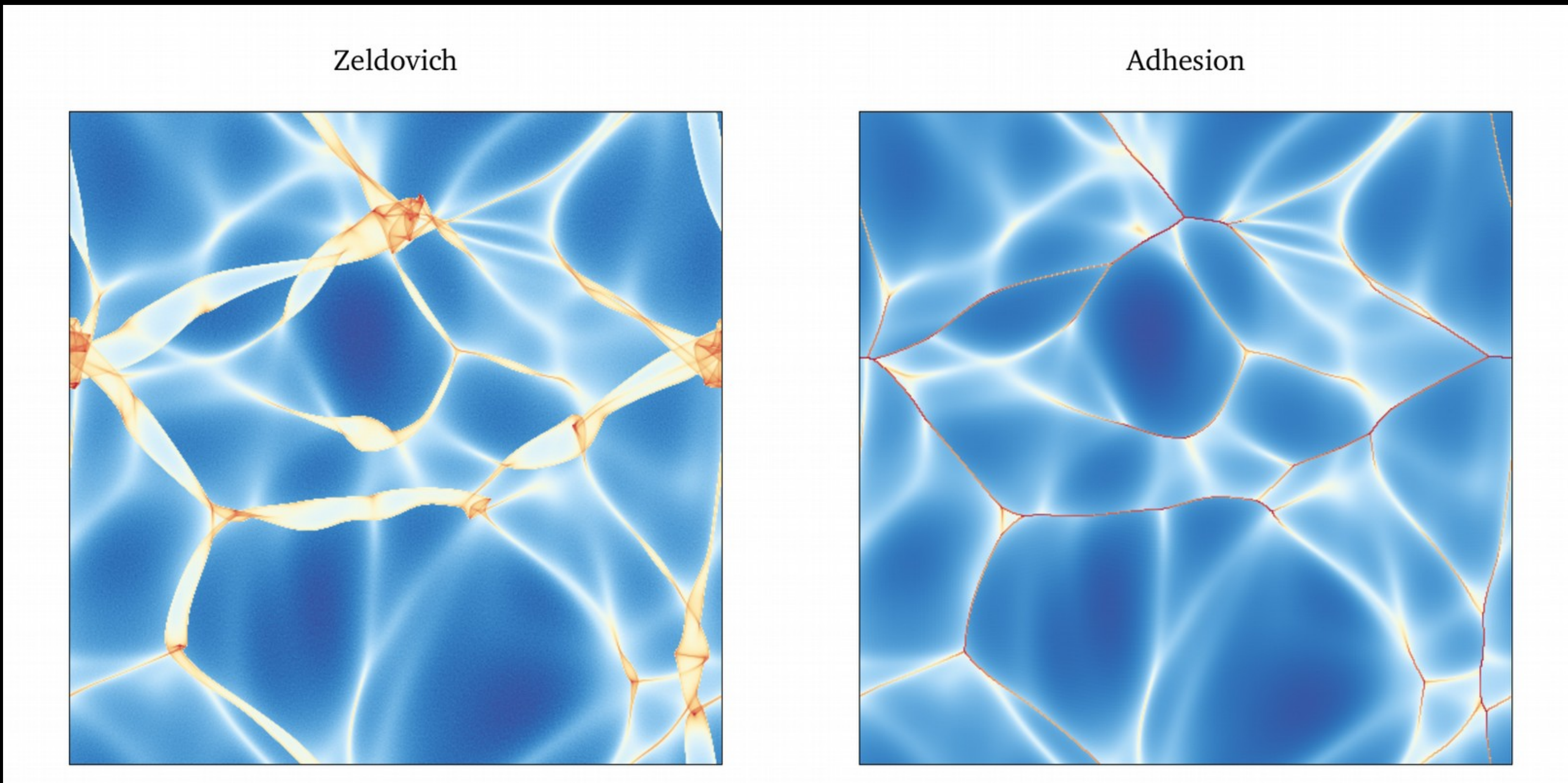
Catastrophe Theory



Catastrophe Theory



Part III: Skeleton of the Web

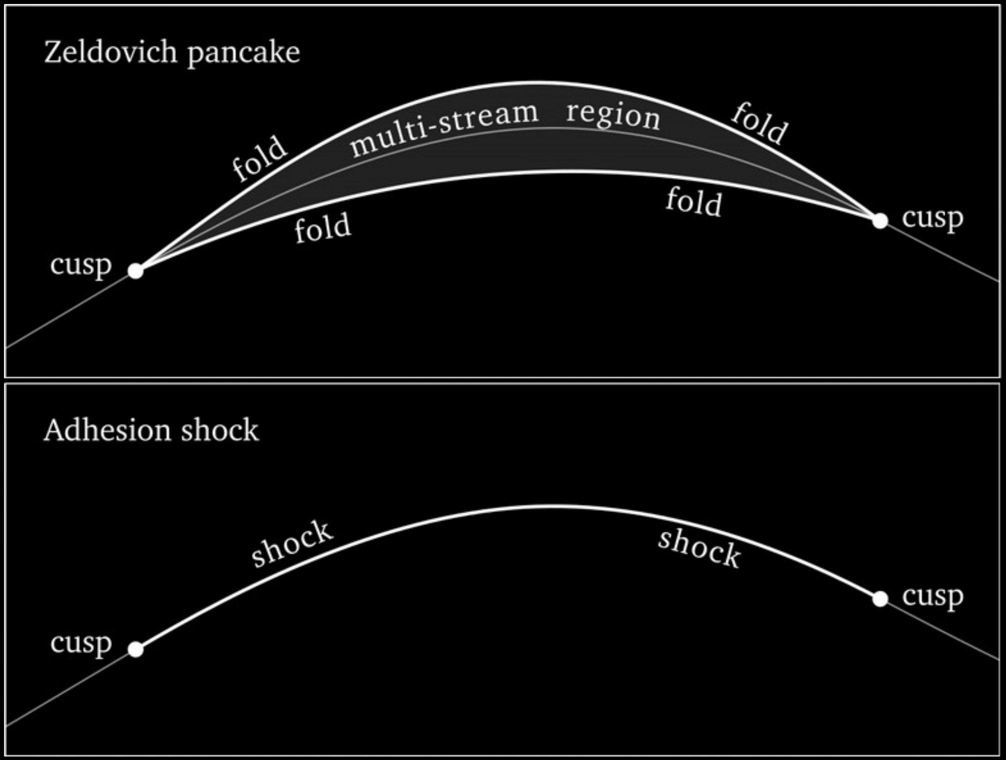


The Adhesion Model

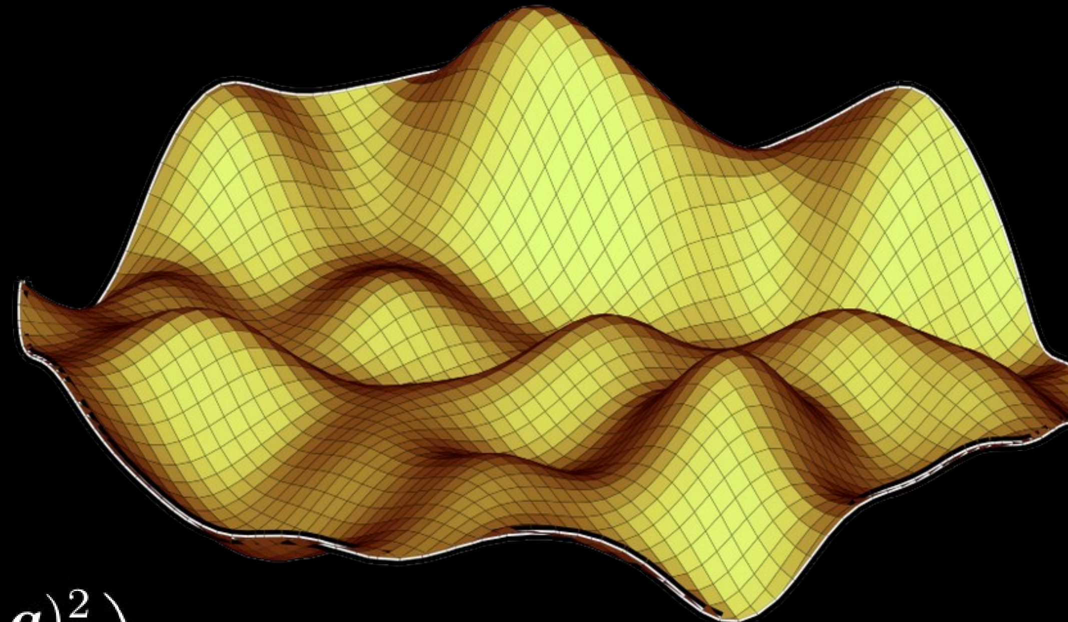
- Add viscosity term to Equation of Motion
- Burgers' equation

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}_x)\, \boldsymbol{v} = \nu \nabla_x^2 \boldsymbol{v}$$

- Solved by E. Hopf (1950)



$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}_x)\, \boldsymbol{v} = \nu \nabla_x^2 \boldsymbol{v}$$

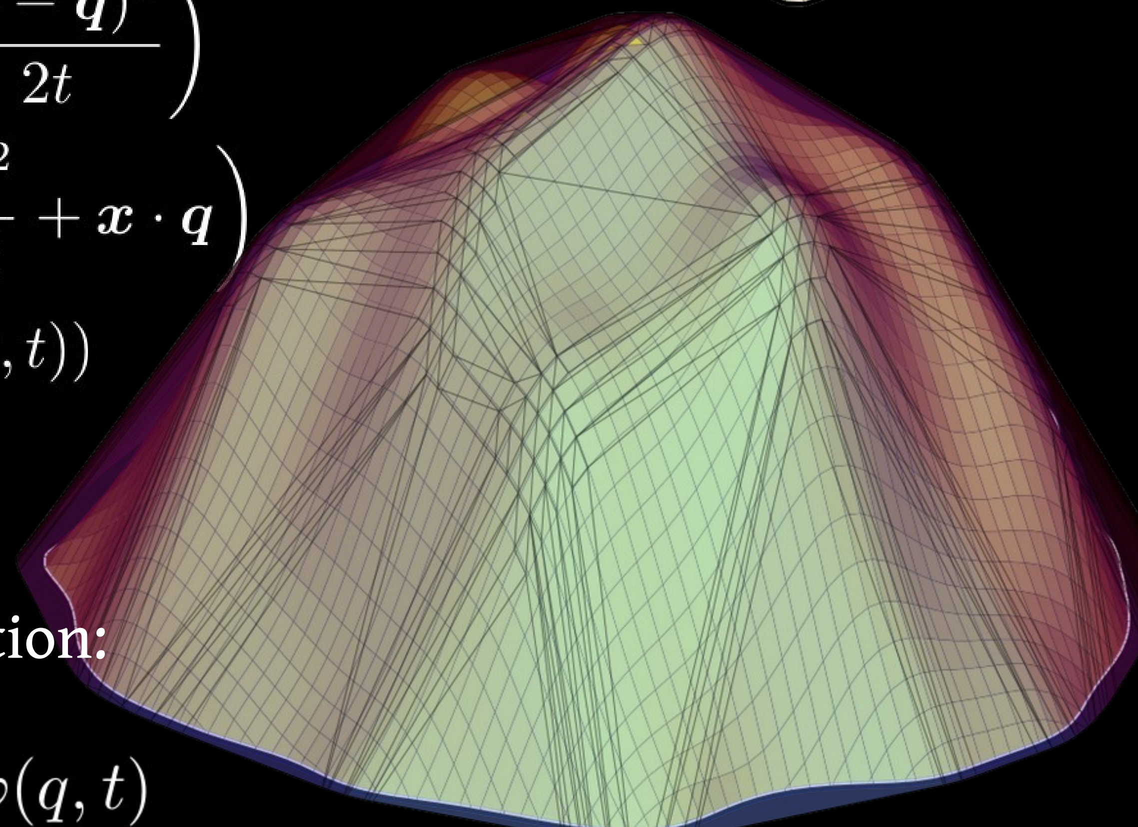


$$\boldsymbol{v} = -\boldsymbol{\nabla}\Phi$$

$$\tilde{\Phi}(\boldsymbol{x}) = \max_q \left(\Phi(\boldsymbol{q}) - \frac{(\boldsymbol{x} - \boldsymbol{q})^2}{2t} \right)$$

$$t\tilde{\Phi}(\boldsymbol{x}) + \frac{x^2}{2} = \max_q \left(t\Phi(\boldsymbol{q}) - \frac{q^2}{2} + \boldsymbol{x} \cdot \boldsymbol{q} \right)$$

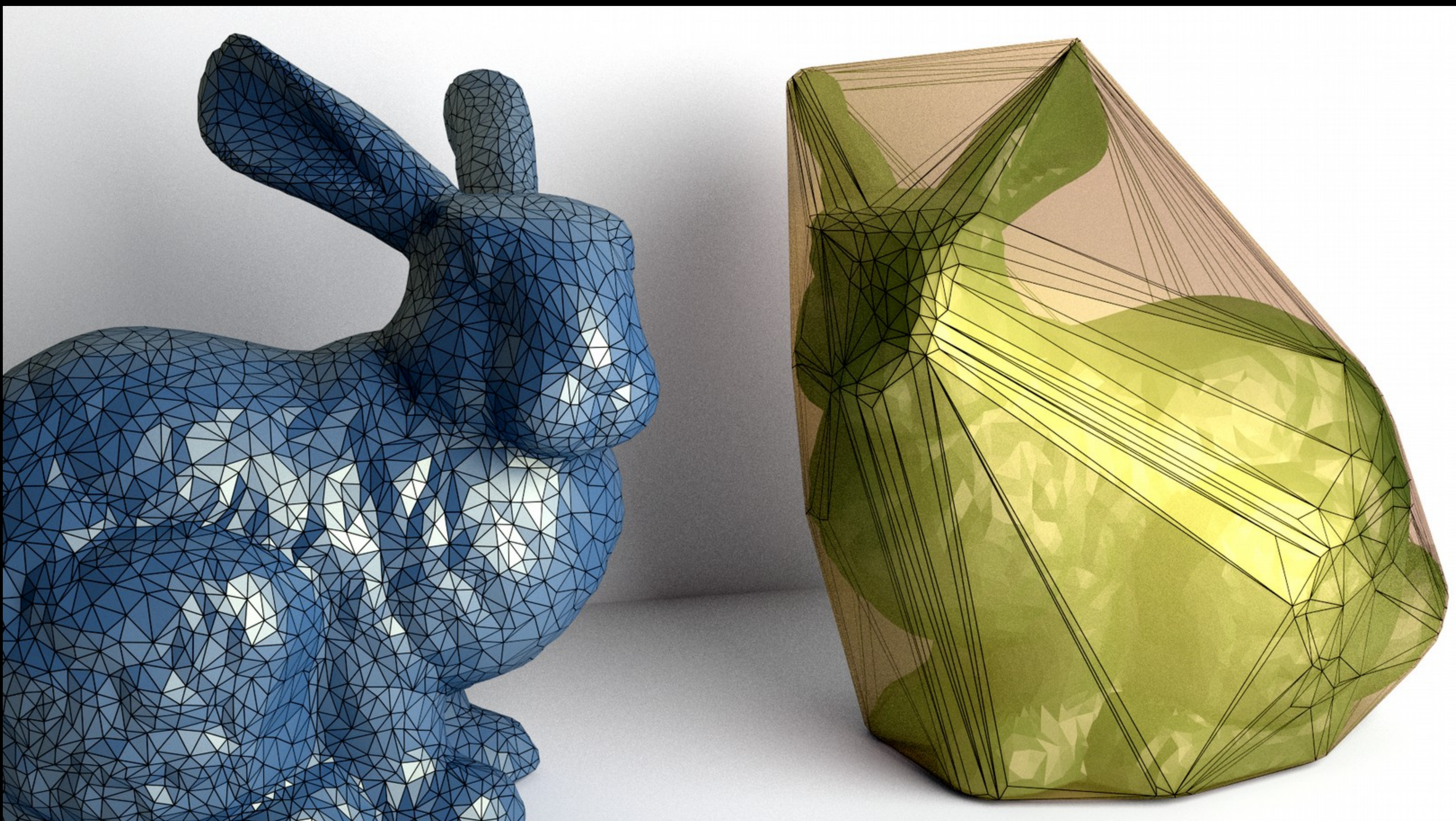
$$= \max_q \left(\boldsymbol{x} \cdot \boldsymbol{q} - \varphi(\boldsymbol{q}, t) \right)$$

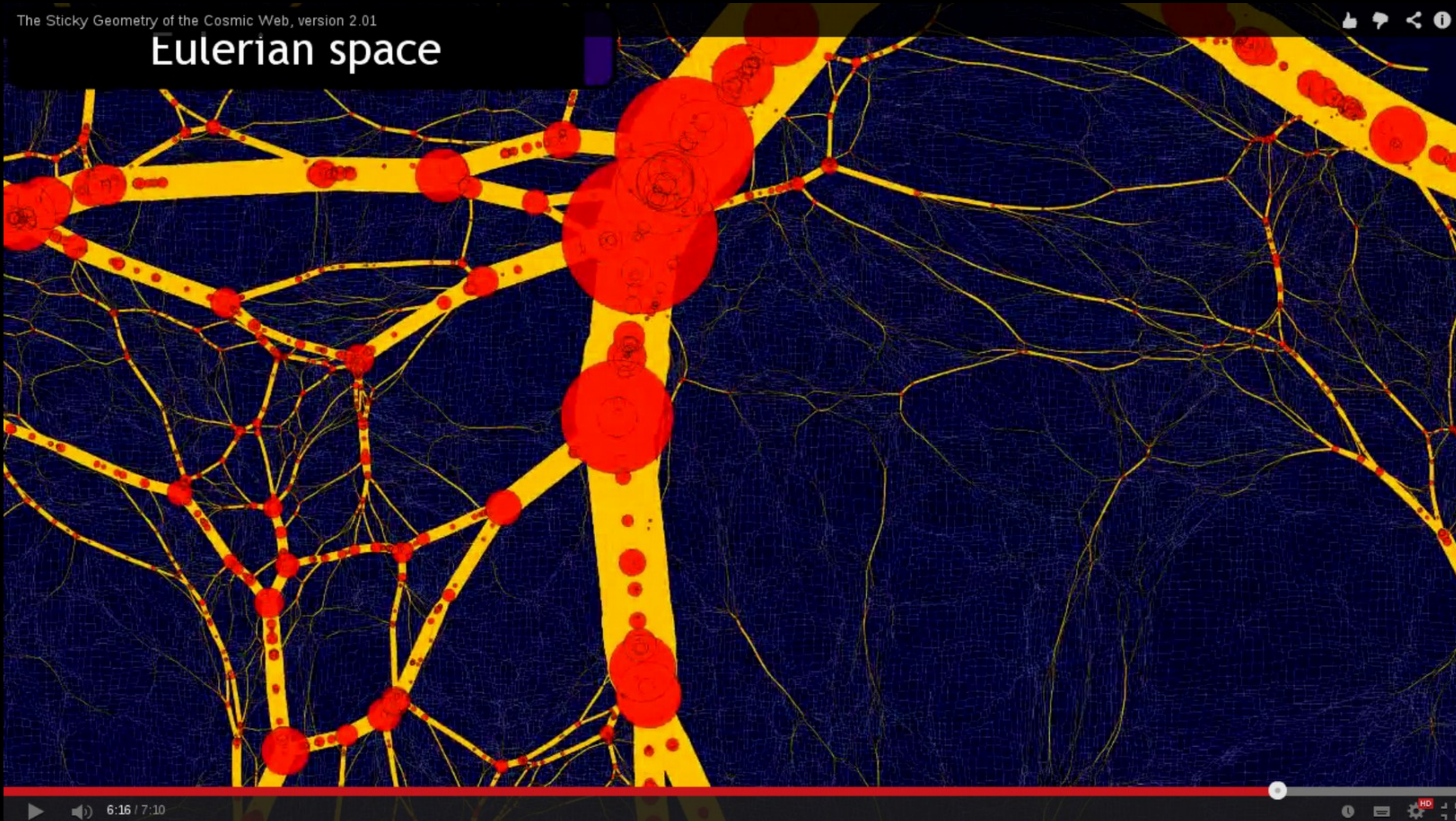


Remember Zeldovich Approximation:

$$\boldsymbol{x} = \boldsymbol{\nabla}_q \left(\frac{q^2}{2} - t\Phi(q) \right) = \boldsymbol{\nabla}_q \varphi(q, t)$$

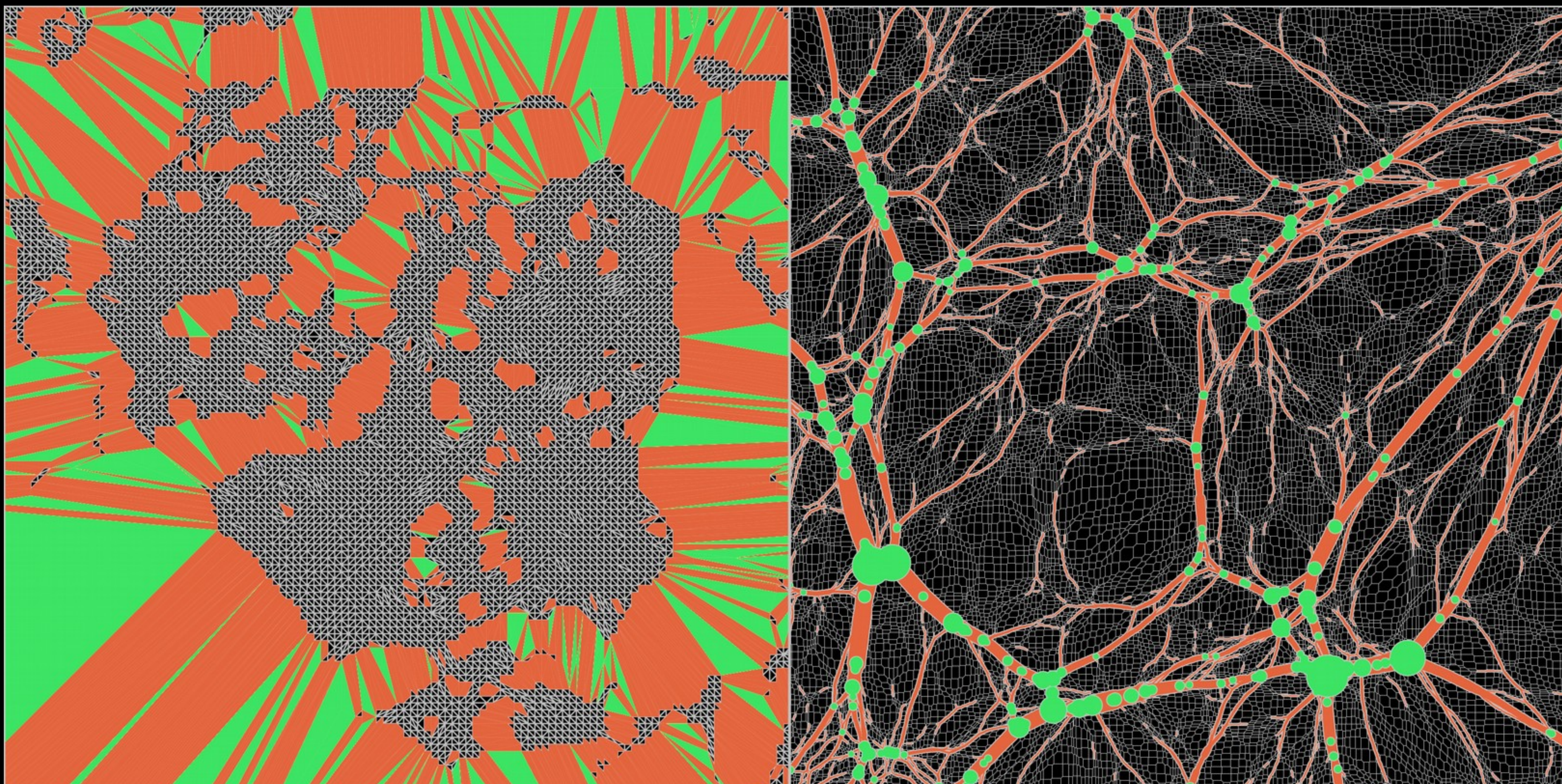
Computational Geometry



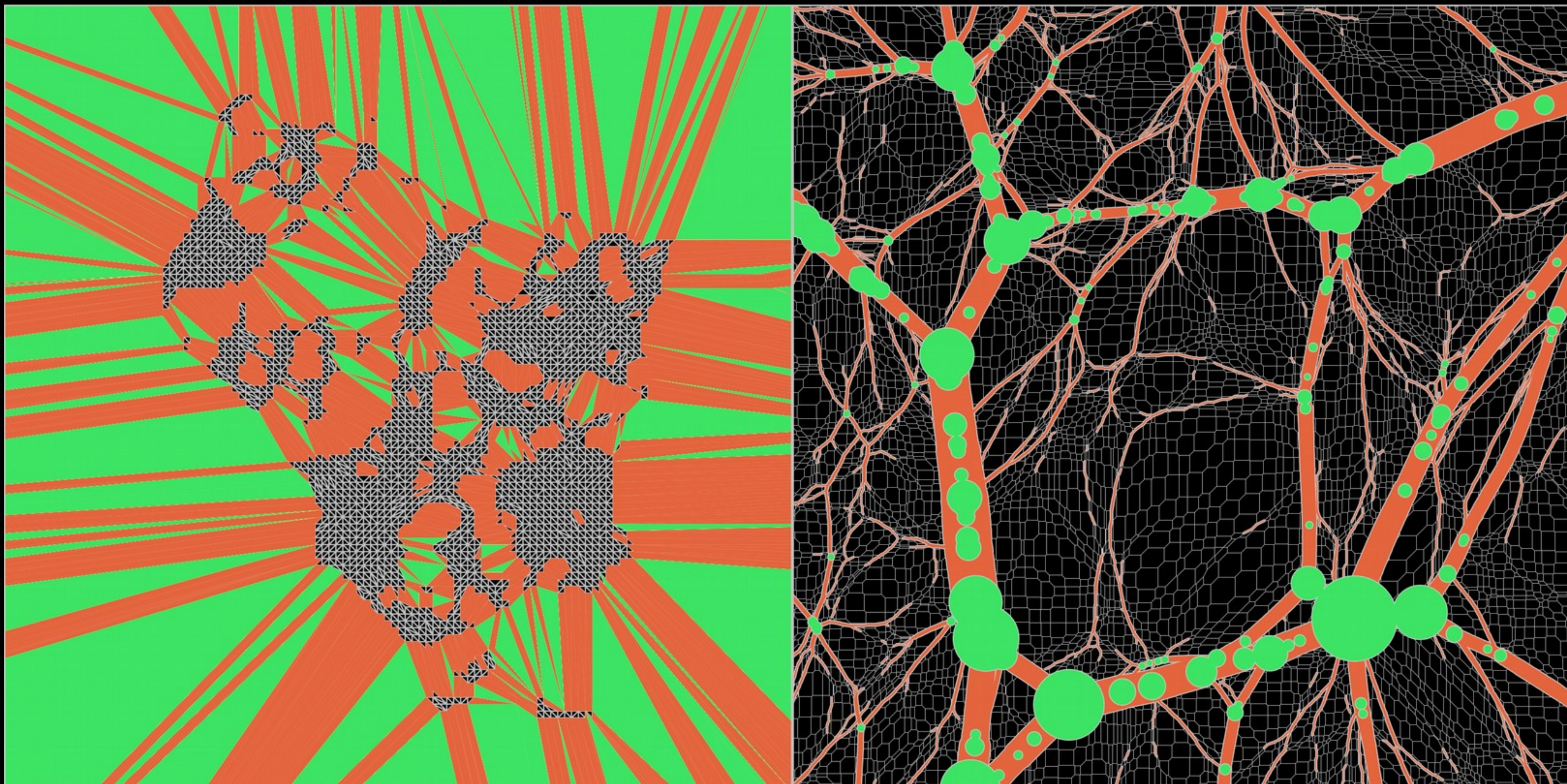


http://youtu.be/wI12X2zczqI
(or search: Geometry of Cosmic Web)

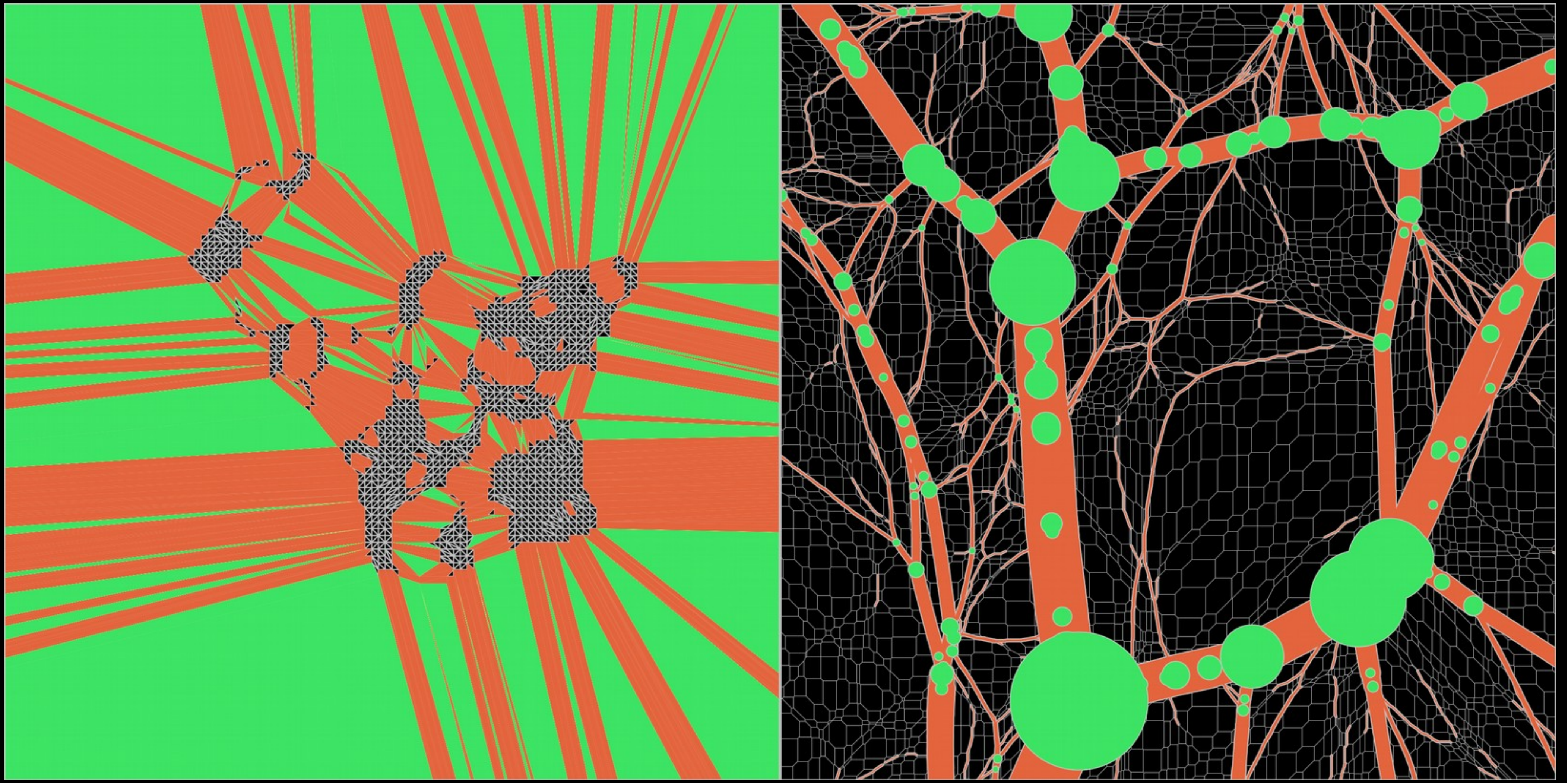
Lagrangian/Delaunay ↔ Eulerian/Voronoi



Lagrangian/Delaunay ↔ Eulerian/Voronoi



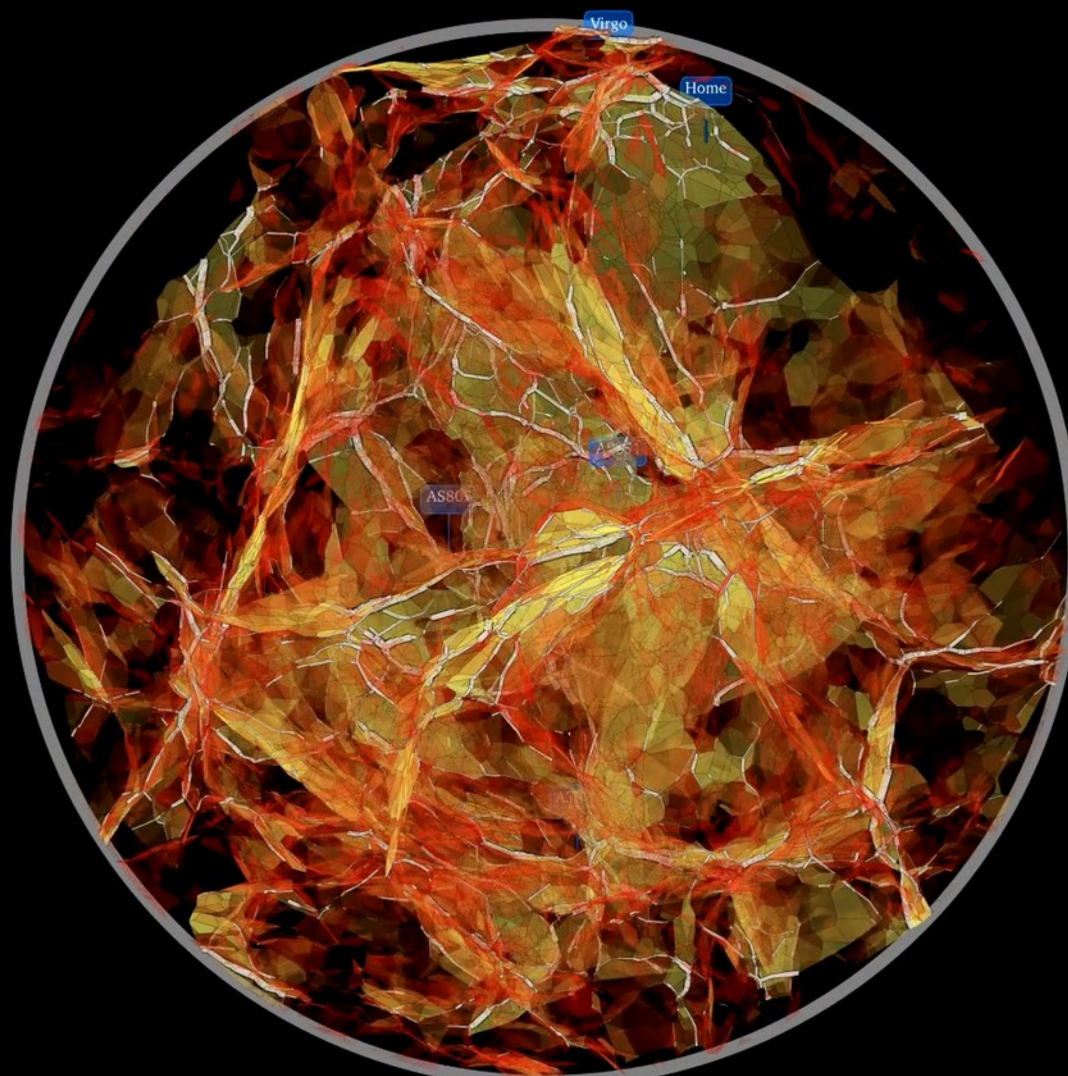
Lagrangian/Delaunay ↔ Eulerian/Voronoi



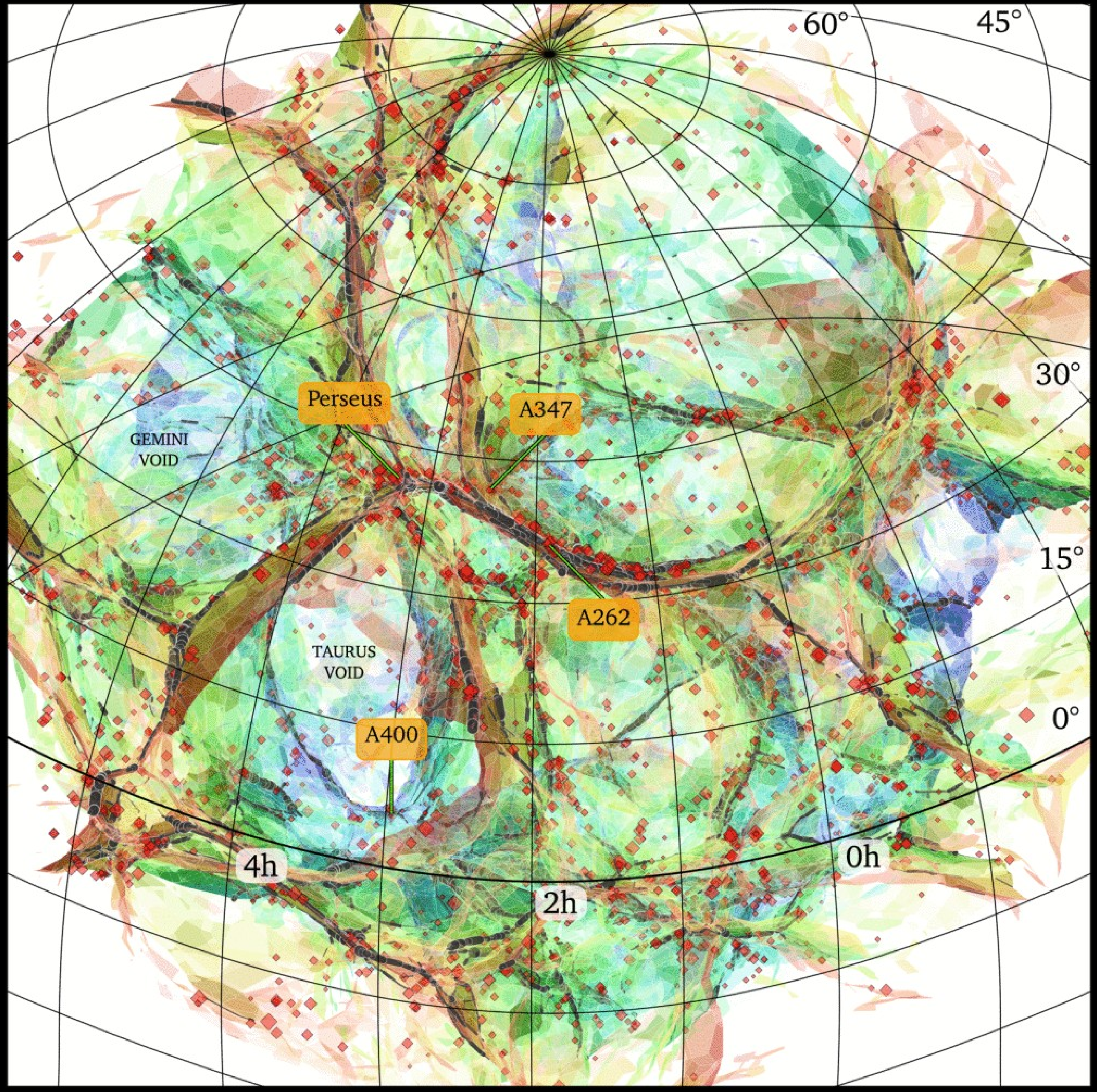
Back to *our* Universe

- Initial conditions from a *reconstruction* based on galaxies in 2MRS (Kitaura 2012, Heß et al. 2014)
- Used CGAL's regular triangulations to find structures

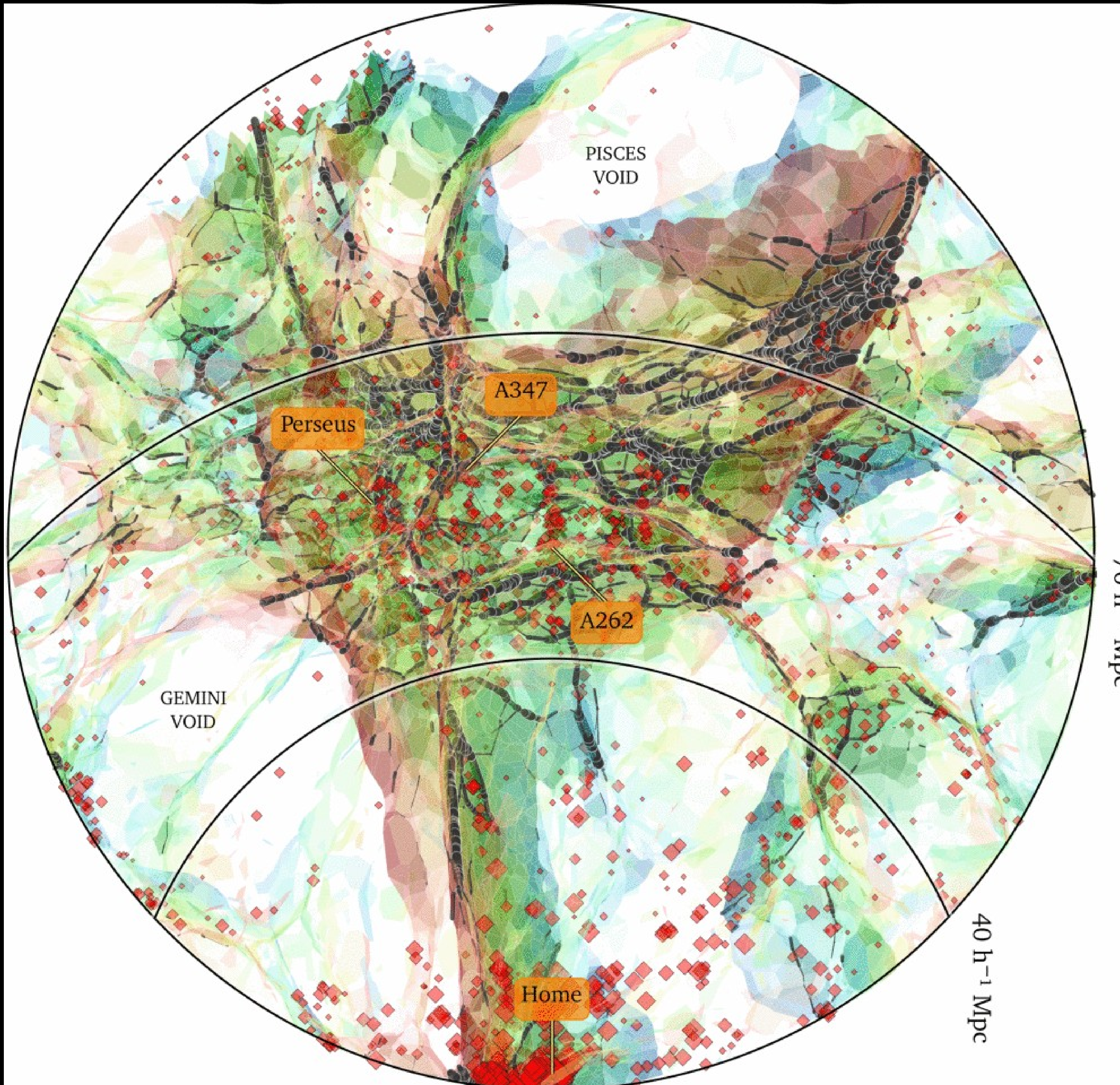
Microscopium



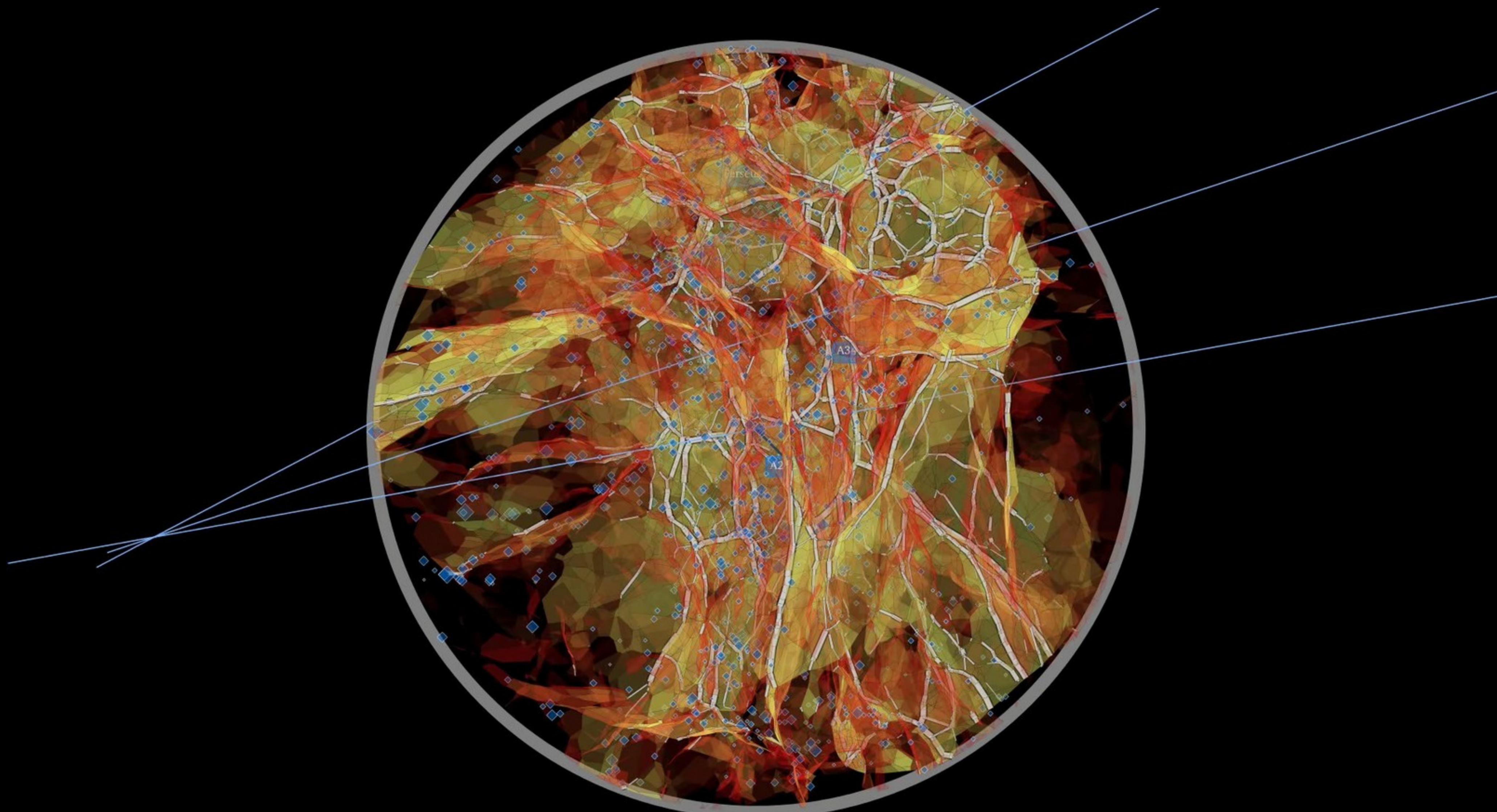
Perseus Pisces

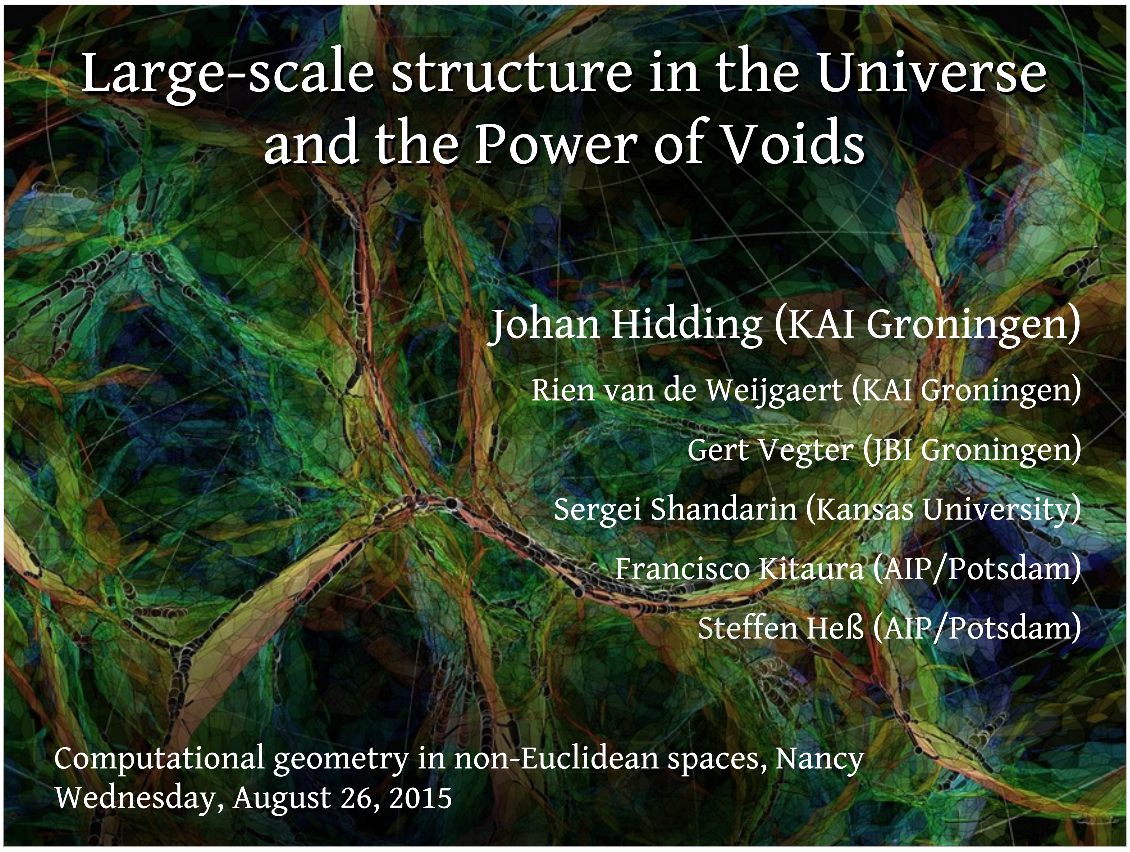


Perseus Pisces

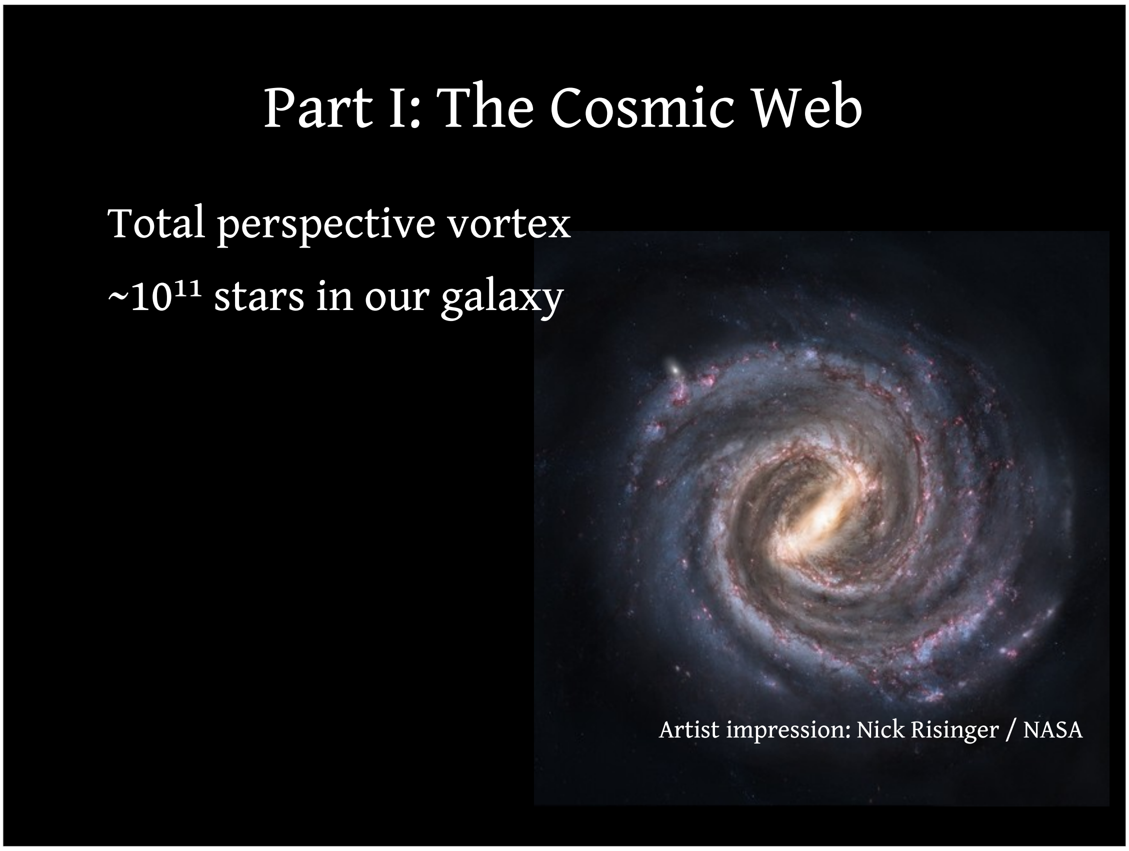


Perseus-Pisces





Thank the organisers...



Total perspective vortex: Douglas Adams, Hitchhikers guide to the Galaxy – torture method: showing people their place in the Universe.

``Space'' it says, ``is big. Really big. You just won't believe how vastly, hugely, mindbogglingly big it is. I mean, you may think it's a long way down the road to the chemist's, but that's just peanuts to space, listen...''

The Total Perspective Vortex derives its picture of the whole Universe on the principle of extrapolated matter analyses. Since every piece of matter in the Universe is in some way affected by every other piece of matter in the Universe, it is in theory possible to extrapolate the whole of creation -- every sun, every planet, their orbits, their composition and their economic and social history from, say, one small piece of fairy cake.

The man who invented the Total Perspective Vortex did so basically in order to annoy his wife.

Trin Tragula -- for that was his name -- was a dreamer, a thinker, a speculative philosopher or, as his wife would have it, an idiot. She would nag him incessantly about the utterly inordinate amount of time he spent staring out into space, or mulling over the mechanics of safety pins, or doing spectrographic analyses of pieces of fairy cake.

``Have some sense of proportion!'' she would say, sometimes as often as thirty-eight times in a single day.

And so he built the Total Perspective Vortex, just to show her. Into one end he plugged the whole of reality as extrapolated from a piece of fairy cake, and into the other end he plugged his wife: so that when he turned it on she saw in one instant the whole infinity of creation and herself in relation to it.

To Trin Tragula's horror, the shock completely annihilated her brain; but to his satisfaction he realized that he had proved conclusively that if life is going to exist in a Universe of this size, then the one thing it cannot have is a sense of proportion.

Part I: The Cosmic Web

Total perspective vortex

~10^{11} stars in our galaxy

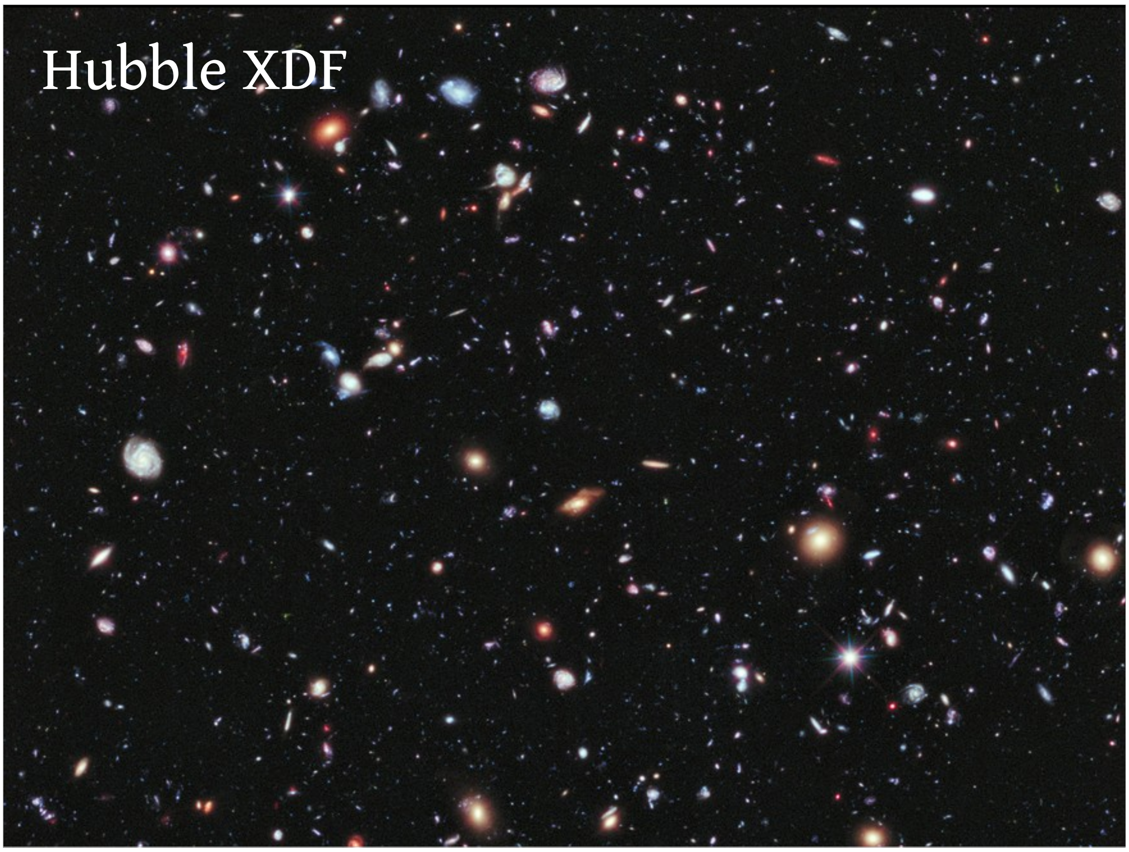
~10^{11} galaxies in the visible Universe

Ordinary matter (stars, planets, people) is only 4% of energy budget

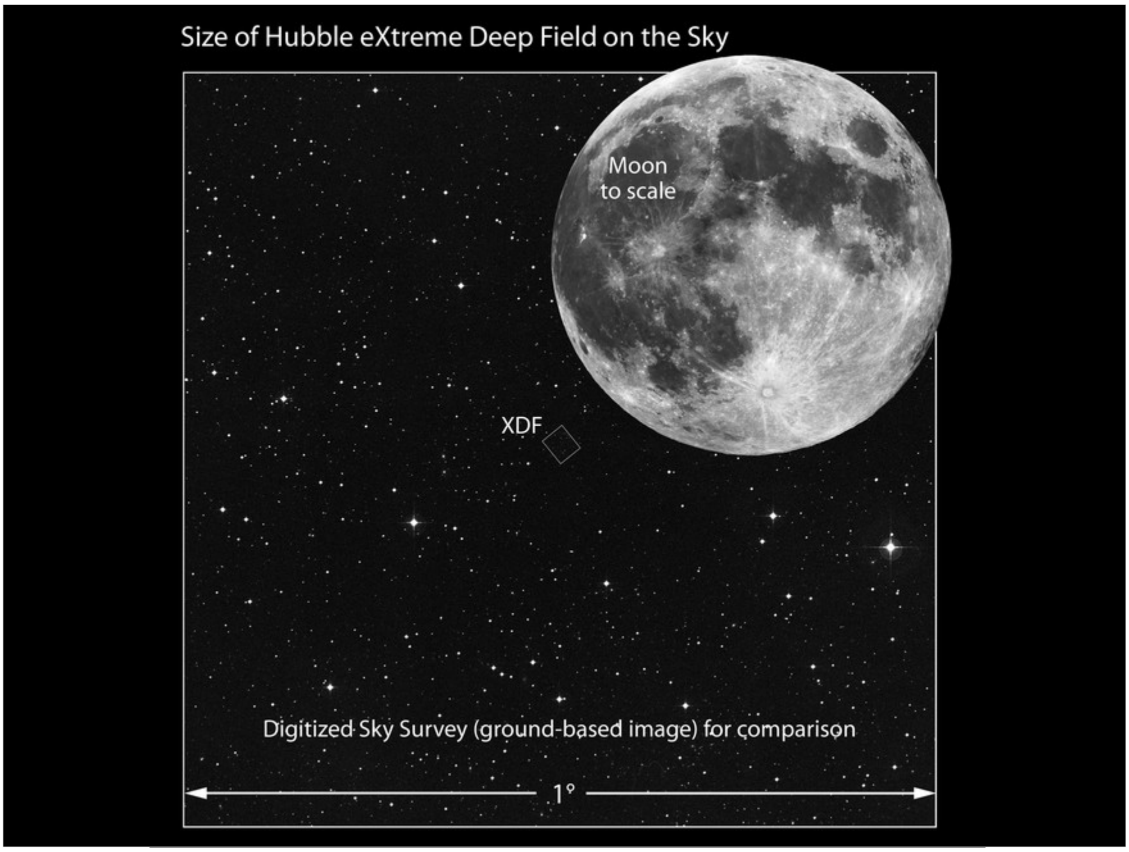
This is the Andromeda Galaxy, we'll measure distances in Mpc (million parsec), that's about 3.26 mln lightyear or 3×10^{19} m.

The distance to Andromeda is ~1 Mpc.

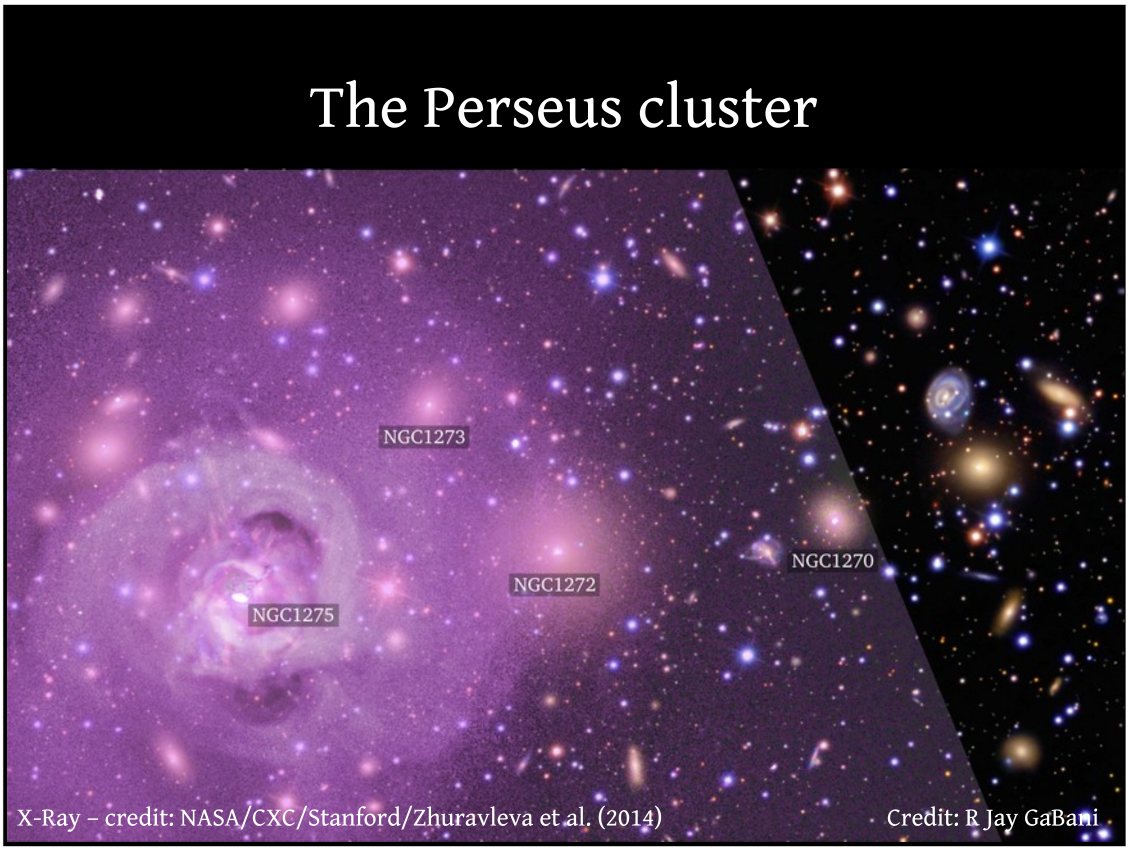
Explaining everything about dark energy or dark matter lies outside the scope of this talk: wikipedia is your friend.



Almost every dust speck is a galaxy. This is only a tiny portion of the sky, as we can see in the next slide.



The little rectangle shows the extend of the image in the previous slide, compared to the area of the full moon.

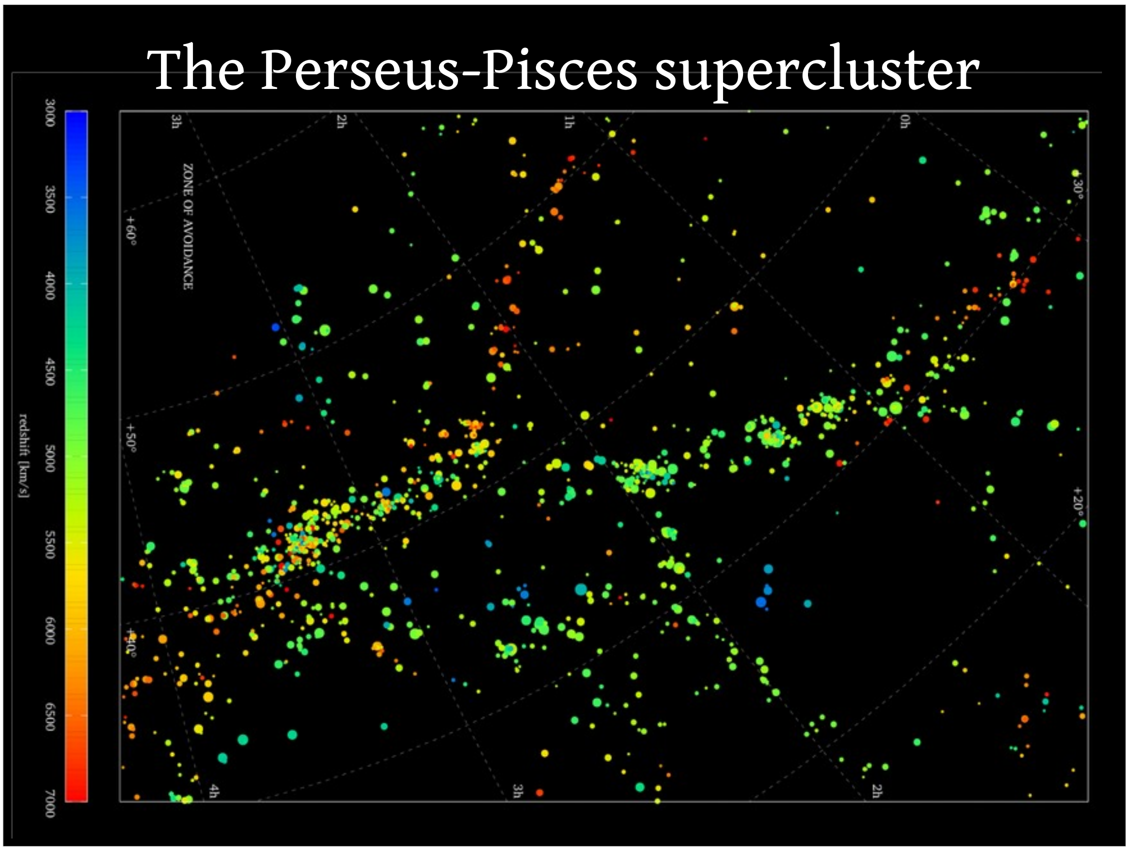


This is the Perseus cluster of galaxies, one of the larger concentrations of galaxies in the neighbourhood, lying at a distance of *only* 50 Mpc.

Most of the mass in this image is not in stars. It is ionized gas that we can only detect in X-ray (here shown in purple)

This image spans about 20' on the sky, at that distance, ~ 1 Mpc

The Perseus cluster is embedded in an even larger structure →

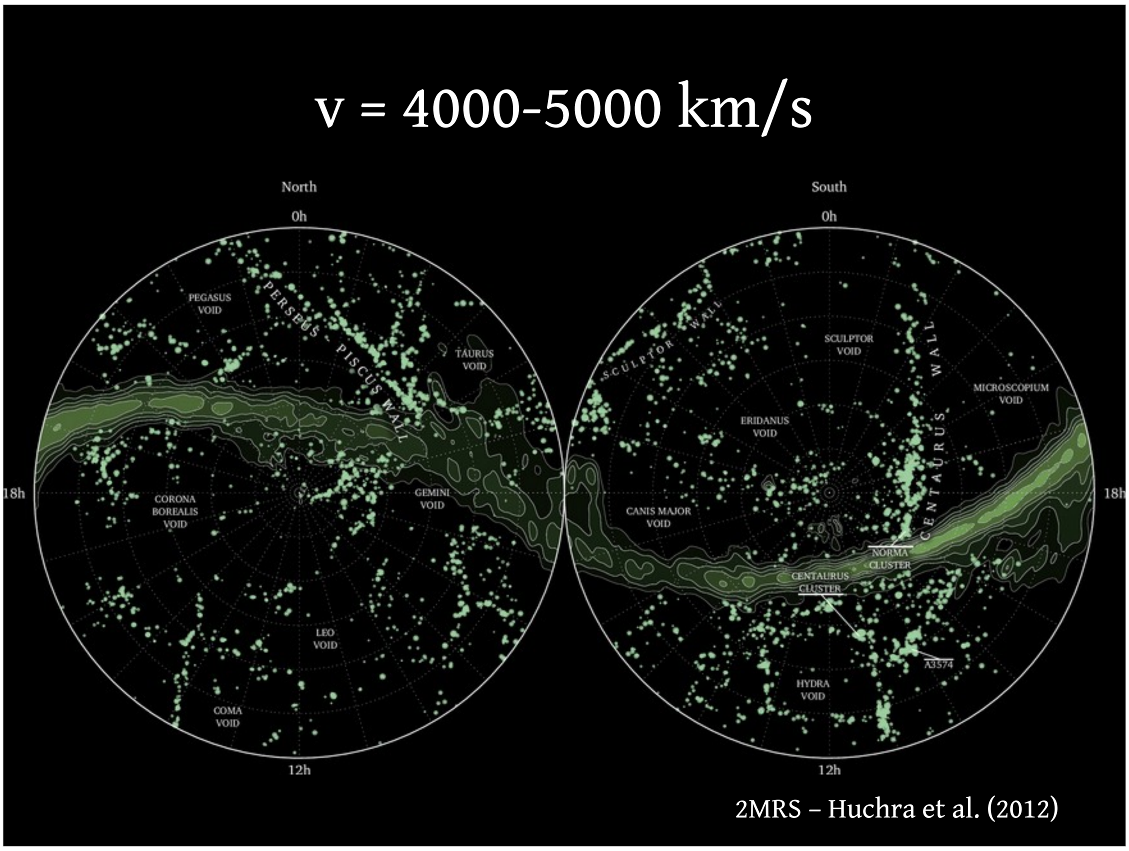


Here we see that galaxies are not randomly distributed but are structured in a filamentary web.

The colours give the recession velocity of each galaxy. This is proportional to the distance of the galaxy: $d = v / H$, where $H = 70$ km/s/Mpc is the famous Hubble parameter.

Note the empty regions: voids

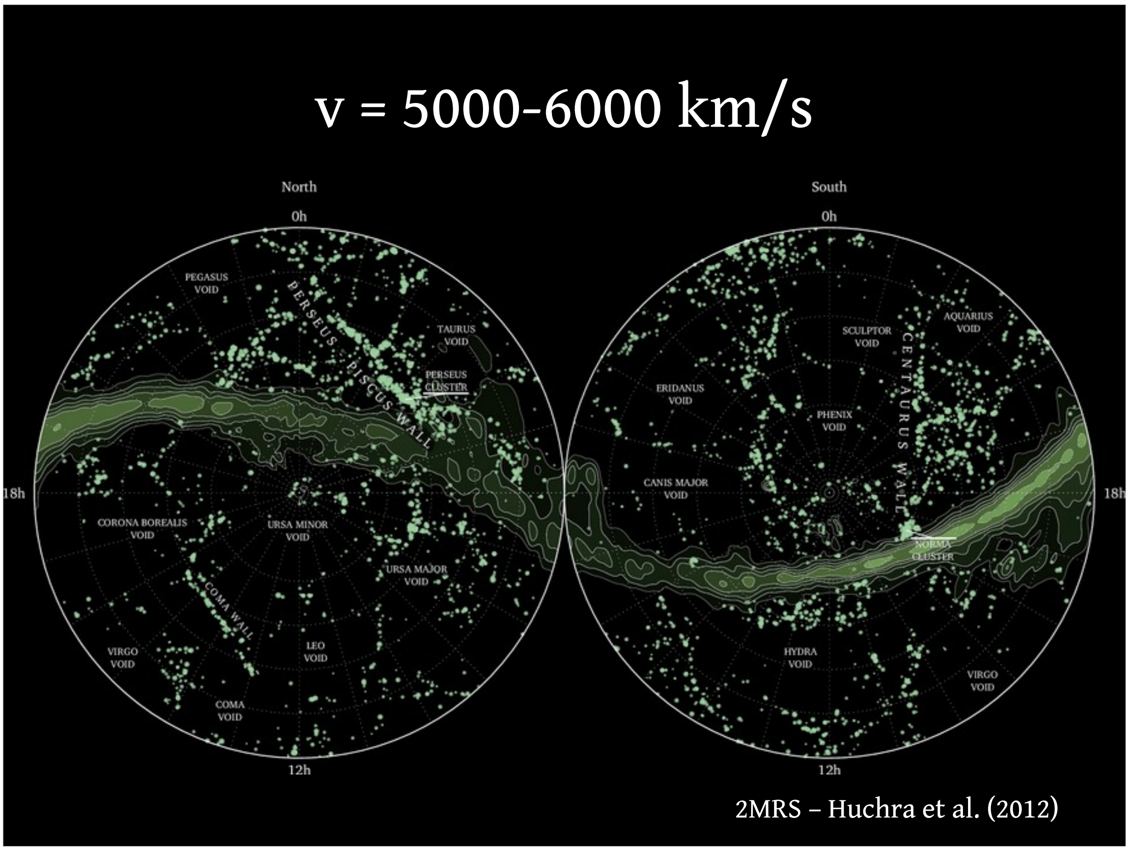
Chalenge: can we describe astrophysical objects in the context of their environment?



This is an equatorial map of the full sky distribution of galaxies that have a recession velocity between 4000 and 5000 km/s. This represents a spherical shell around home. The contours show the places where dust in our own Milky Way (as well as stars) makes observation of other galaxies impossible.

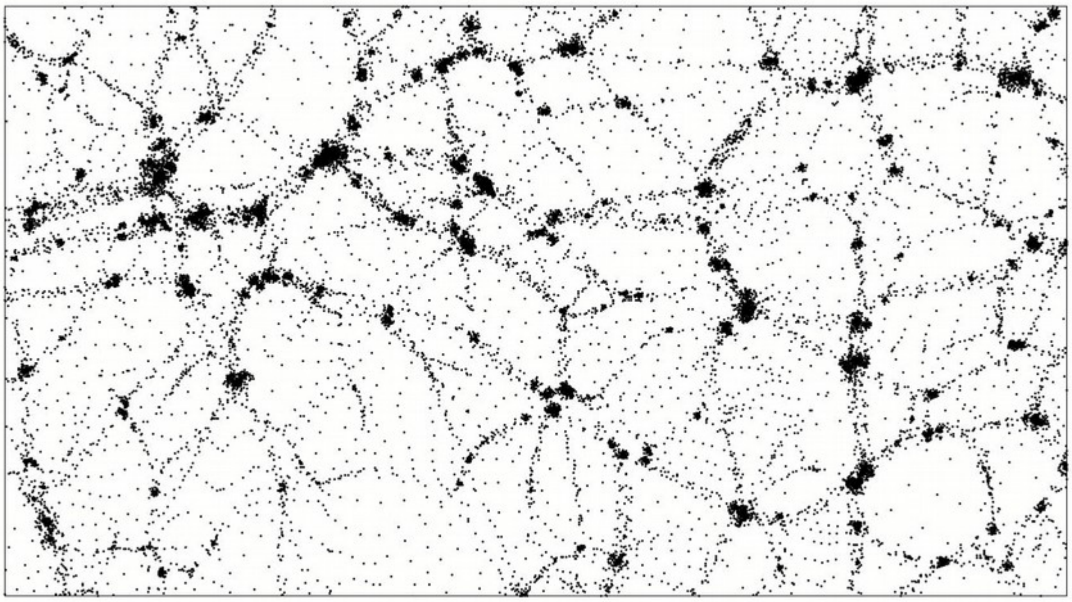
On the southern hemisphere we see the Centaurus Great Wall (actually part of Local Sheet)

On the northern hemisphere the largest structure is Perseus-Pisces, even better visible on next slide.



Similar to the previous slide, just one distance bin further away.

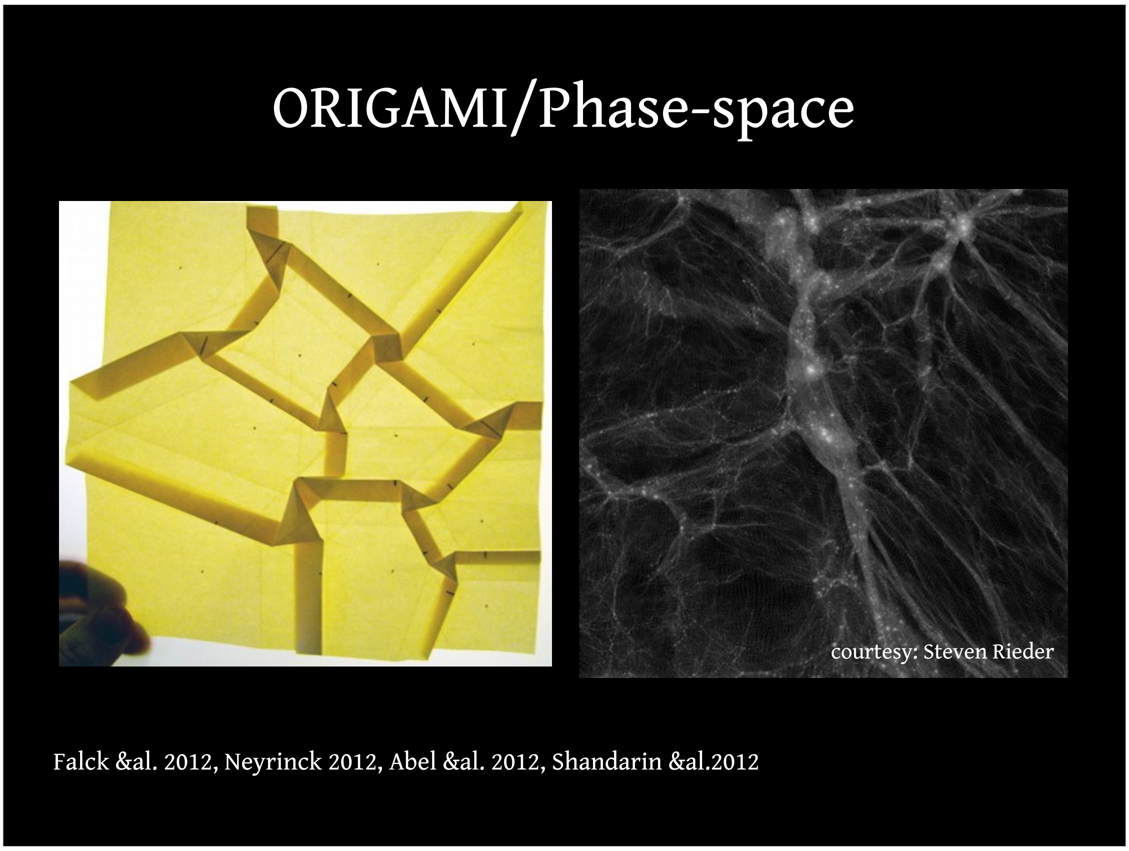
Part 2: Structure formation



Dark matter!!! We simulate the dynamics of dark matter, by having a bunch of particles move in their ever evolving joint gravitational potential field. This movie shows the process in a 2D toy model.

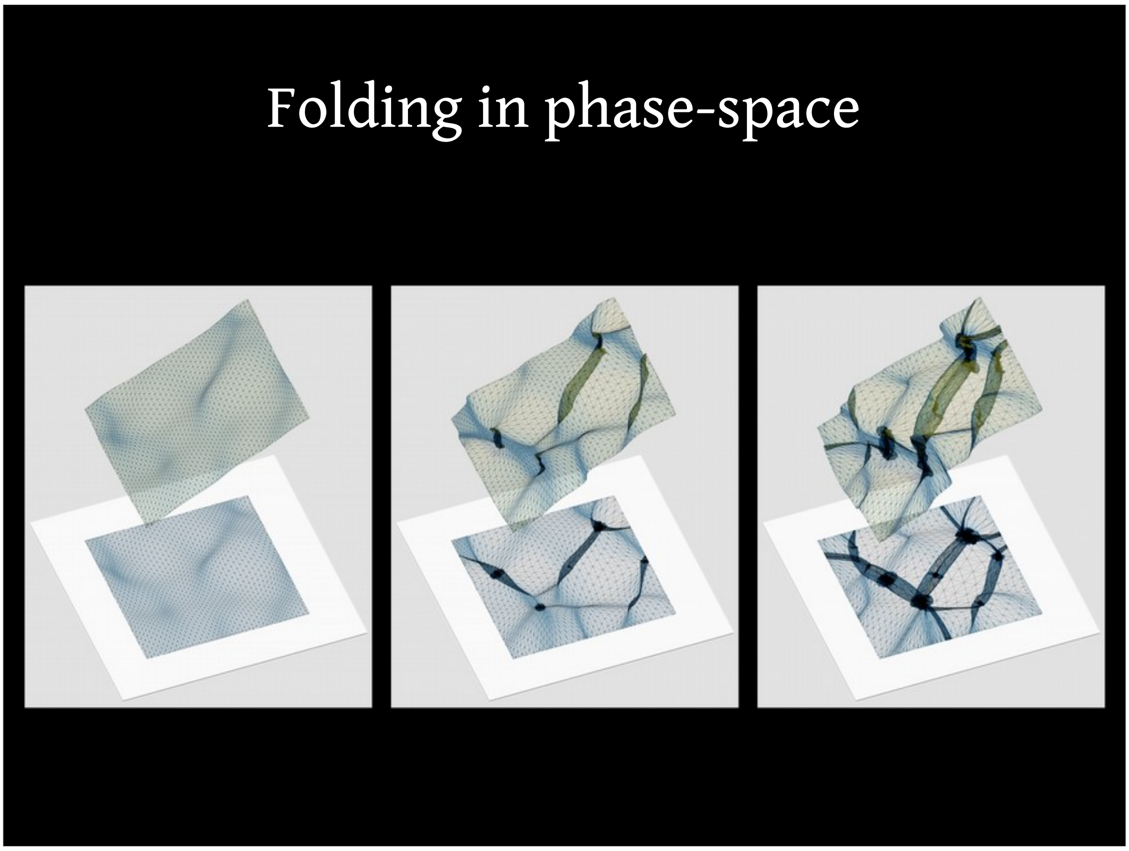
The universe starts mostly homogeneous. Very small fluctuations in the density are then amplified by the process of gravitational collapse.

This is just particles: we want to describe web structures!



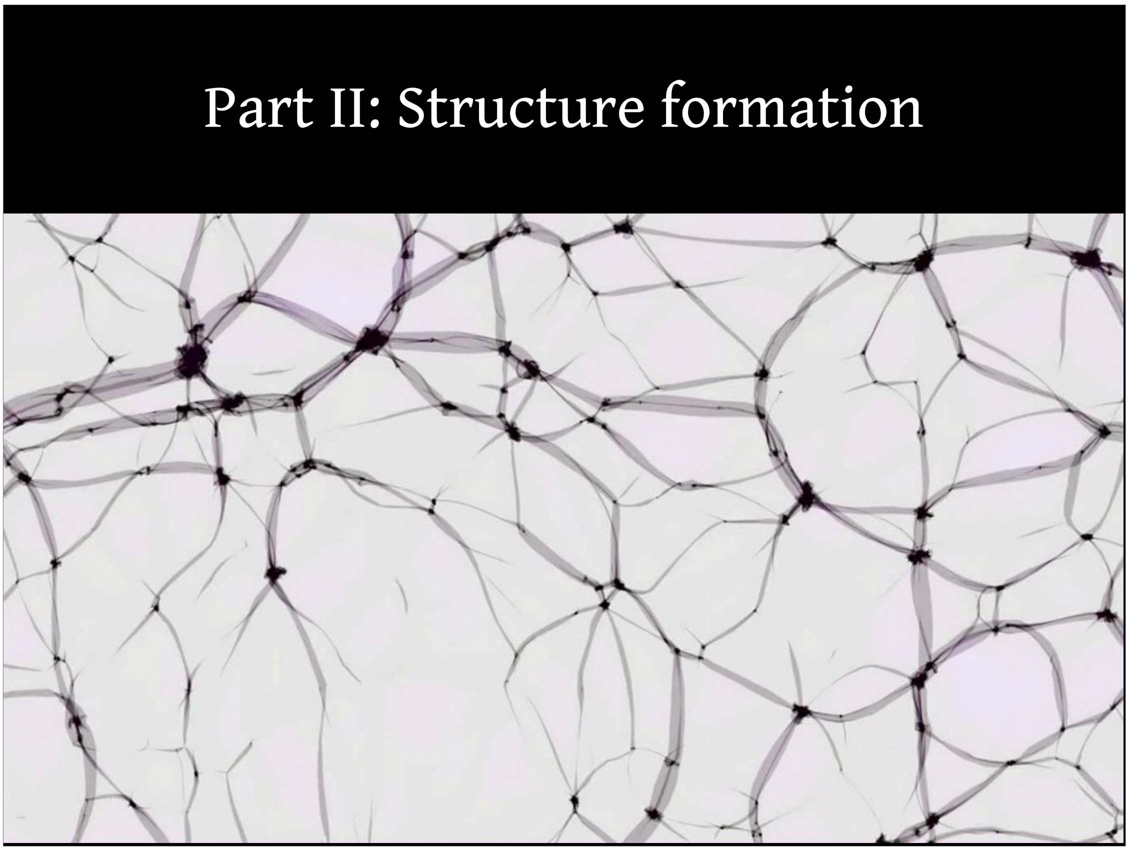
Similarities between these images? On the right we see a snapshot of a realistic simulation. Note the sharp boundaries of the filaments: we can recognise caustics here.

These caustics can be understood by representing the growth of structure as a sheet that's folding up in a higher dimensional space.

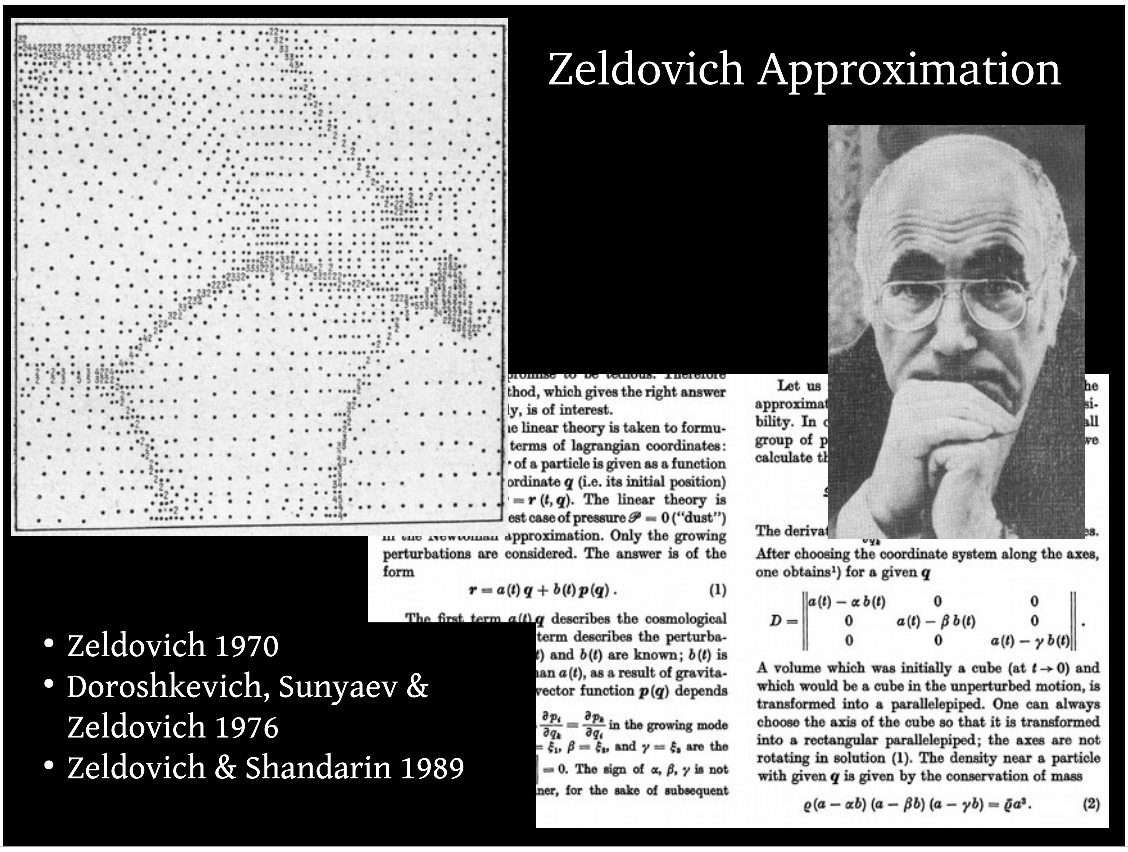


This sheet starts flat: we show here in two dimensions the final configuration (Eulerian space) and one dimension for the original configuration (Lagrangian space). At t=0, tthese configurations are identical and the sheet is flat.

As structures start to grow, the sheet deforms and folds. We can study the emerging structures by studying the folding of the sheet.



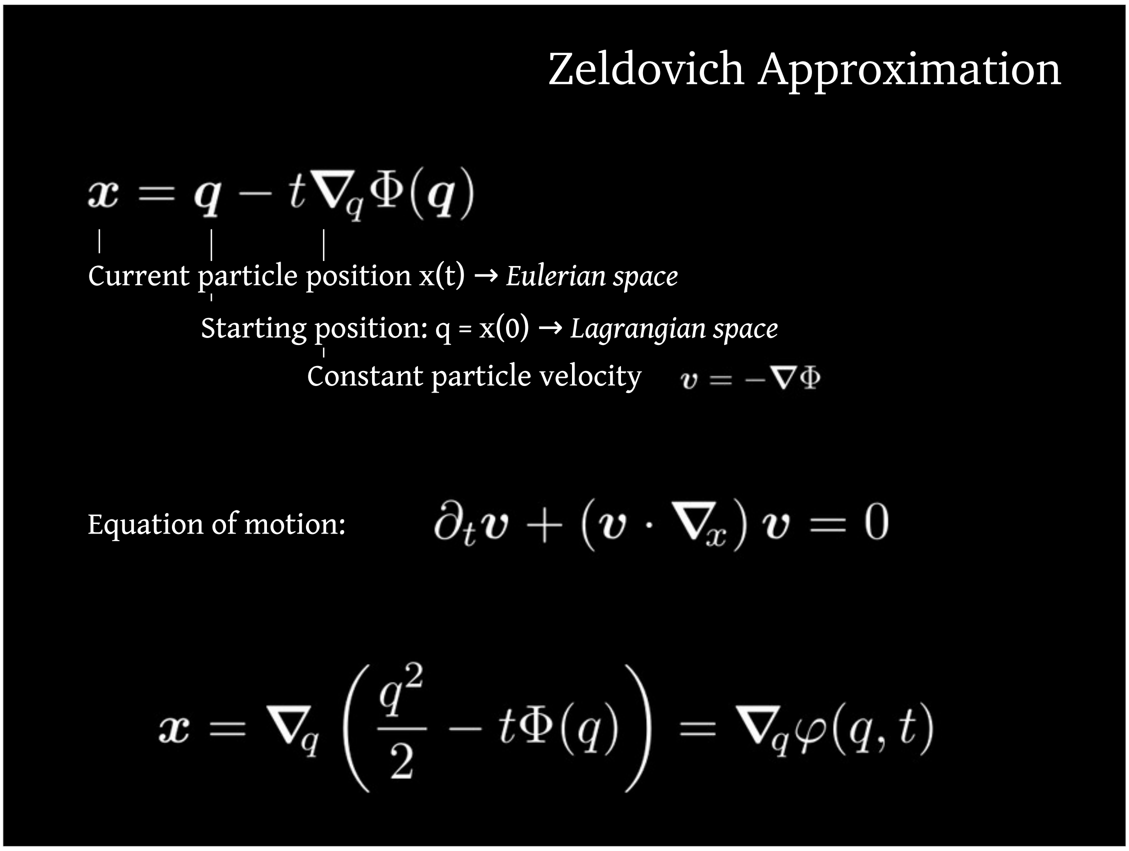
Now we repeat the previous simulation, but fill in the gaps using the phase-space sheet.



To learn about the rules of phase-space folding we use an approximation scheme where things are more simple.

Renowned cosmologist Zeldovich, famous for the Katouchka and the Hydrogen Bomb, retired to do cosmology.

The image shows the earliest rendering of the Zeldovich approximation.



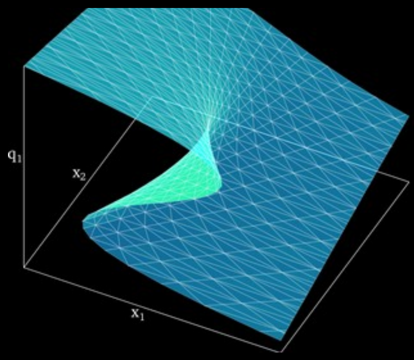
First order approximation: give particles a kick and let them fly. After some time particle trajectories start to cross. The approximation loses formal validity. Zeldovich genius lies in pushing the approximation beyond this point.

The approximation takes the form of a map L: q → x, mapping particles from their original position (Lagrangian space) to their current position (Eulerian space).

This approximation can be arrived at from first principles, starting from Newton equations of motion on an expanding background: see Shandarin & Zeldovich 1989 for a review.

How do we compute the density?

Caustic formation → Pancakes!



$$\mathcal{L} : \boldsymbol{q} \mapsto \boldsymbol{x}$$

$$\boldsymbol{x}(\boldsymbol{q}, t) = \boldsymbol{q} - t\boldsymbol{\nabla}_q \Phi(\boldsymbol{q})$$

$$\rho(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = \langle\rho\rangle\mathrm{d}\boldsymbol{q}$$

$$\begin{aligned}\frac{\rho(\boldsymbol{x})}{\langle\rho\rangle} &= \sum_{q^\star \in \mathcal{L}^{-1}(x)} \left|\frac{\partial x_i}{\partial q_j}\right|^{-1}_{q=q^\star} \\ &= \sum_{q^\star \in \mathcal{L}^{-1}(x)} \begin{vmatrix} 1 - t\alpha(\boldsymbol{q}) & 0 & 0 \\ 0 & 1 - t\beta(\boldsymbol{q}) & 0 \\ 0 & 0 & 1 - t\gamma(\boldsymbol{q}) \end{vmatrix}^{-1}_{q=q^\star}\end{aligned}$$

Sheet described by map from Lagrangian to Eulerian coordinates.

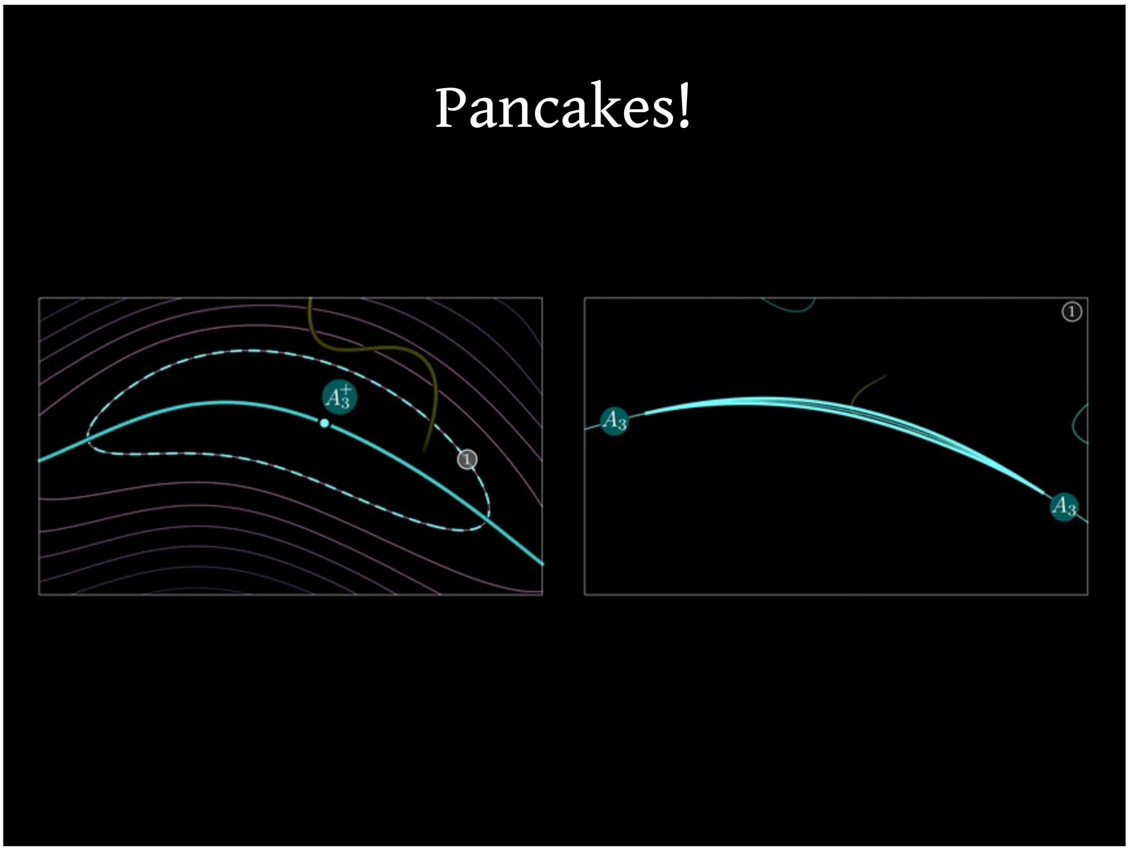
The sheet starts flat, then deforms and folds.

Eigenvalues show when matrix becomes singular.

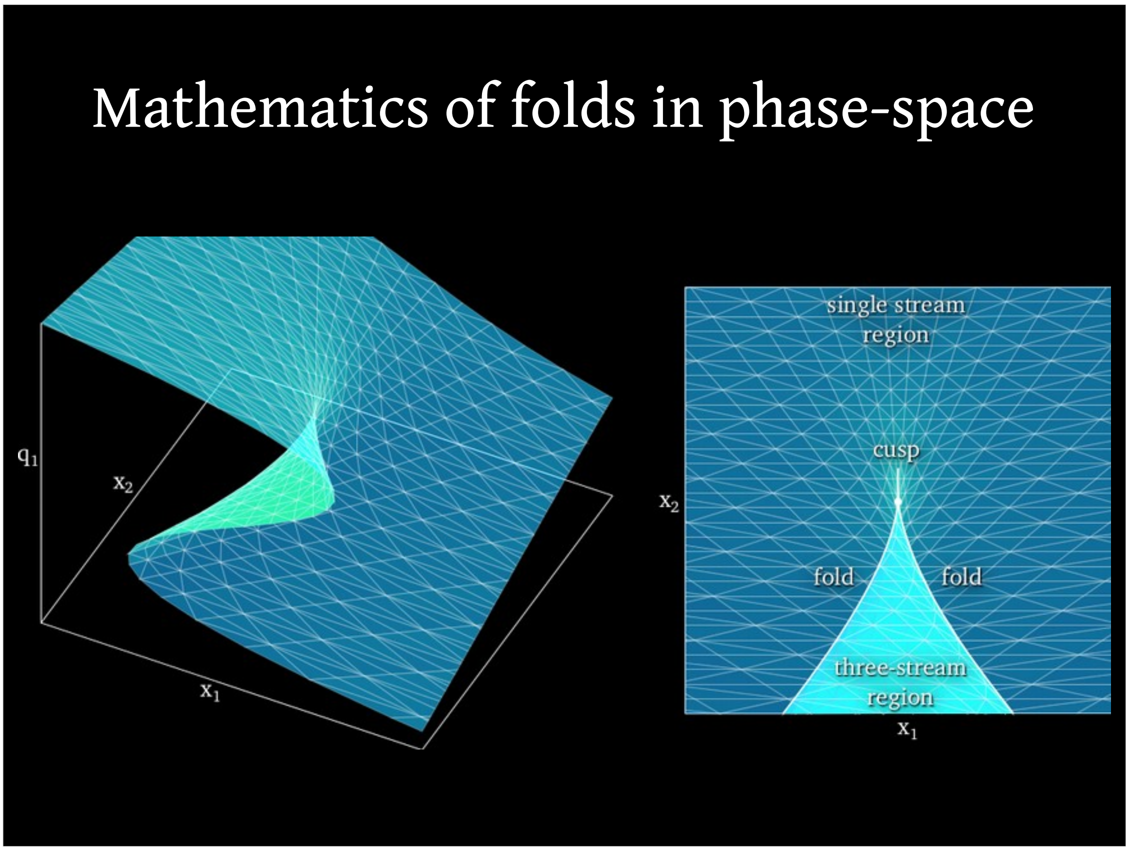
The density is then $\rho/\langle\rho\rangle = 1/(1-t\alpha)(1-t\beta)(1-t\gamma)$

This means we have a singularity in the projection whenever $t = 1/\alpha$, $t = 1/\beta$ or $t = 1/\gamma$.

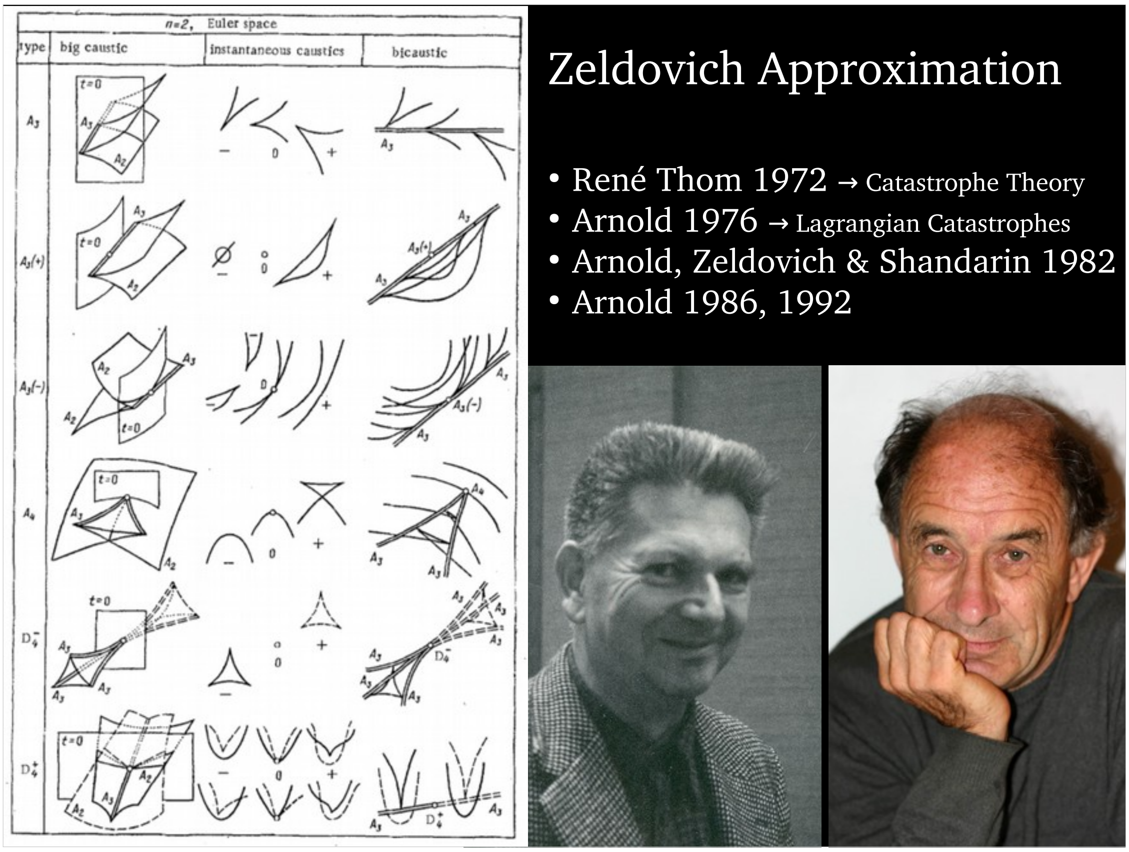
Structure formation starts at maxima of first eigenvalue. Level sets of the eigenvalues become the principal elements to study the evolution of structures in the Zeldovich Approximation. *Note that these are the eigenvalues of the Hessian of Φ*.



Pancakes are formed! Here we show the level sets of the first eigenvalue (in 2D) near a maximum. The cyan contour maps to a very thin pancake structure in Eulerian space.



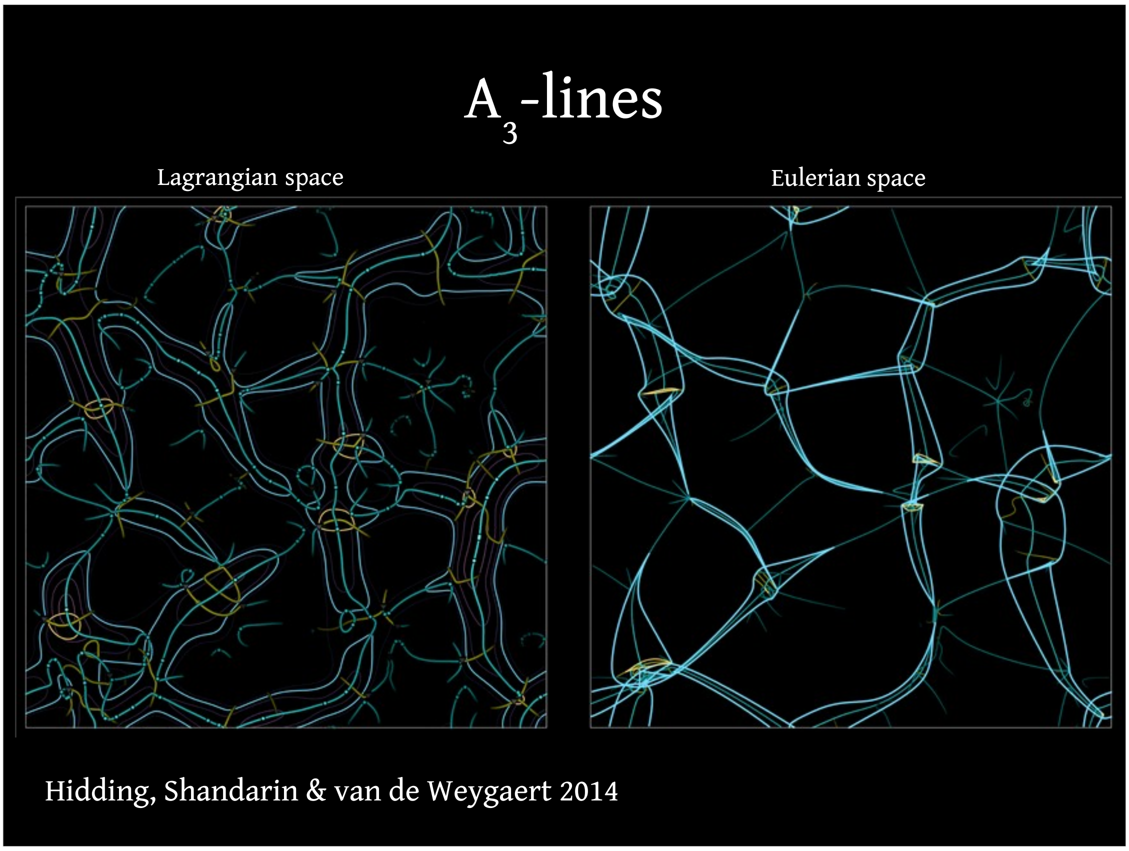
Around the end of a pancake we can distinguish:
– single-stream region
– caustic: two folds an a cusp
– multi-stream region



The mathematics is that of catastrophe theory, first developed by René Thom, later extended to Lagrangian manifolds by Vladimir Arnold.

Arnold was aware of Zeldovich' problem with cosmic structure formation. They wrote a paper explaining catastrophe theory for cosmologists (Arnold, Zeldovich & Shandarin 1982) in 2D. Later Arnold extended it to 3D.

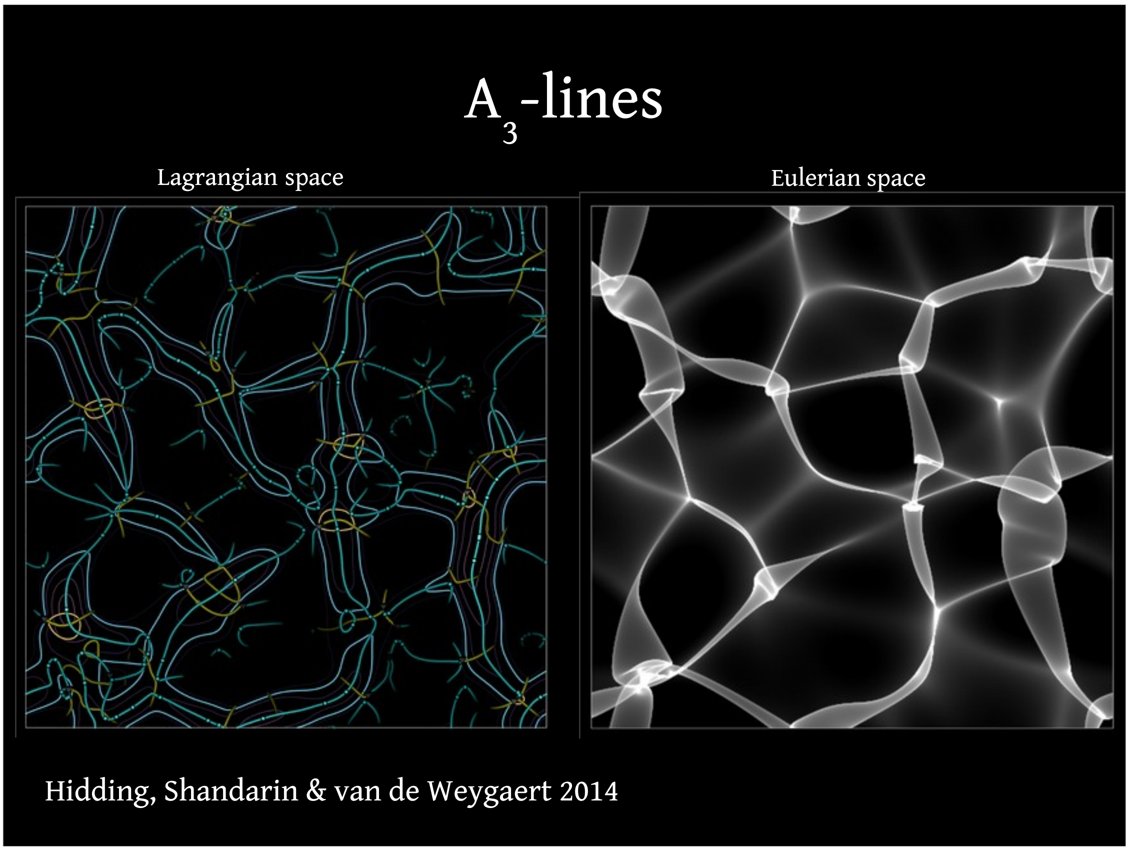
They still did not see how the singularities would fit in a global picture of the formation of the cosmic web. Most of all they lacked computer power and algorithms to visualise the implication of their work.



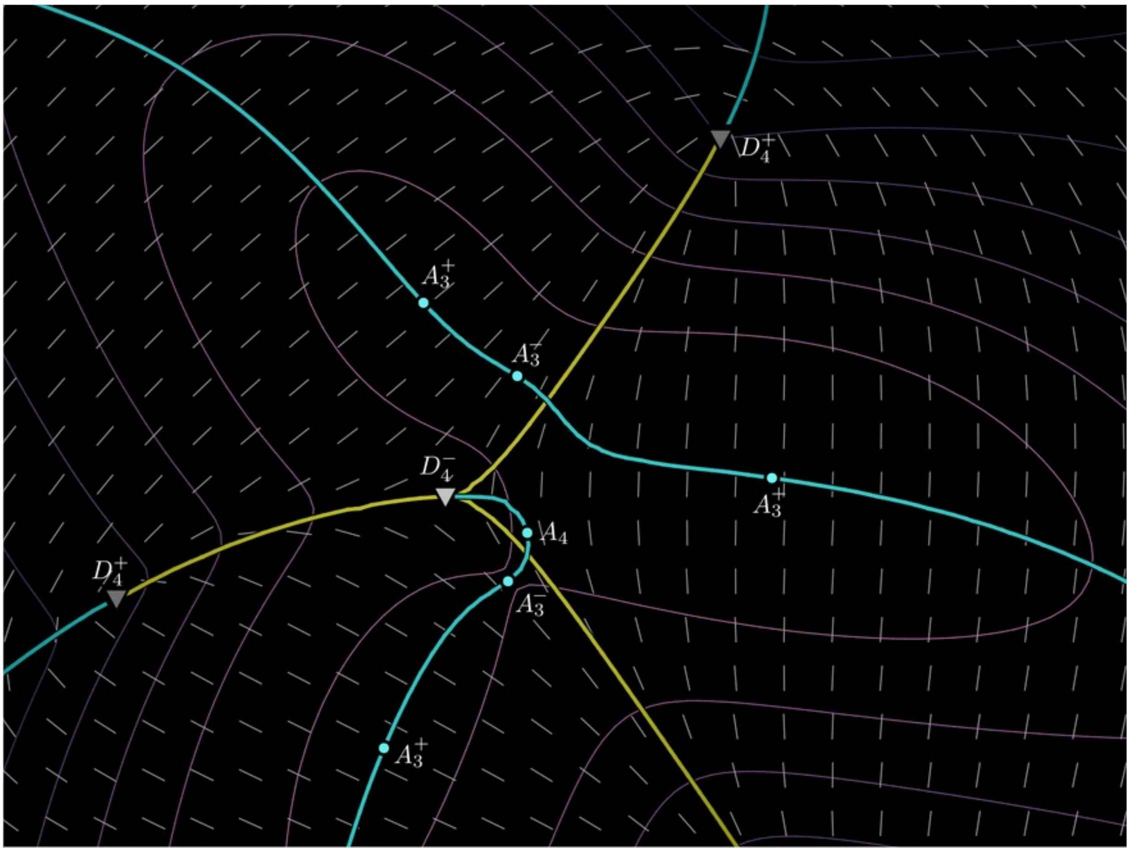
Central entities in studying the metamorphoses (perestroikas) of the caustic network are the A3 lines (greenish cyan). These lines give the blueprint of the structures forming.

The bright cyan lines show a selected contour of the first eigenvalue, mapped to Eulerian space on the right. The cusps (A3 singularity) that we see there correspond to the places where the contour intersects the A3 lines.

The same is shown for the second eigenvalue in orange tones. How do we find A3 lines?



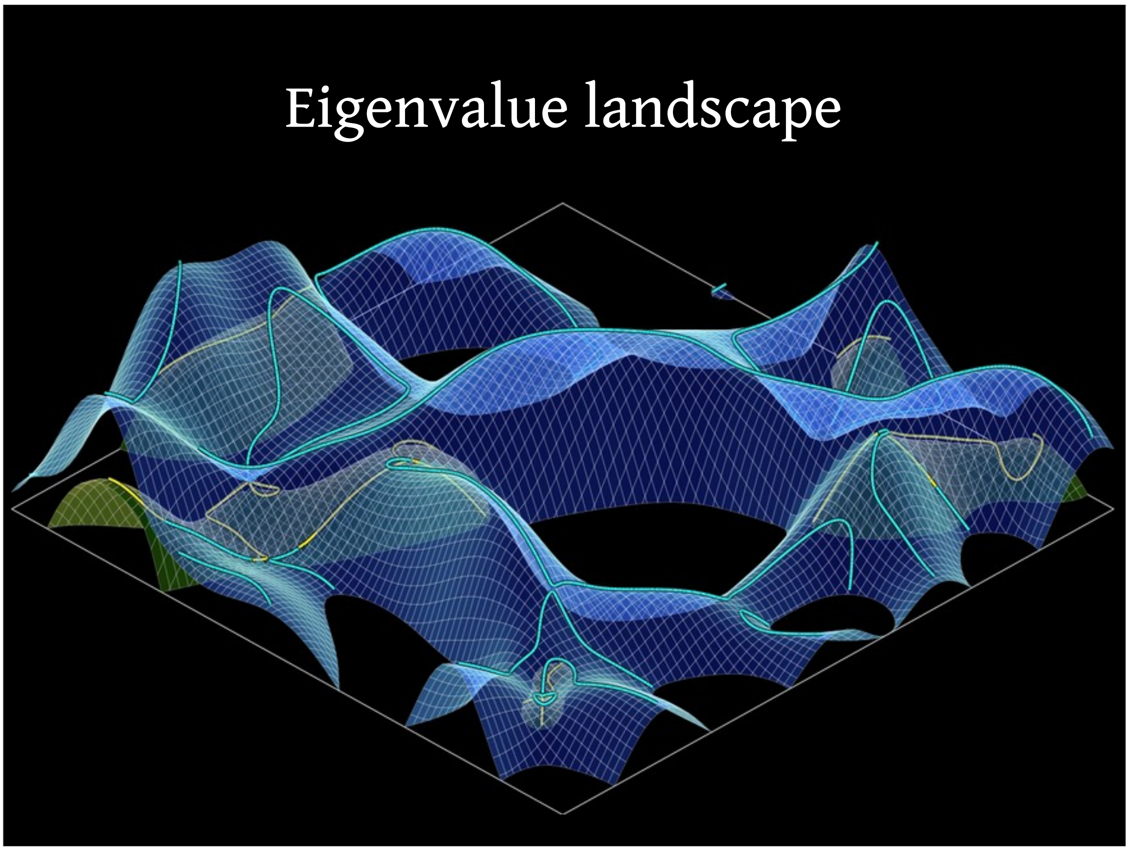
A3-lines
Lagrangian space
Eulerian space
Hidding, Shandarin & van de Weygaert 2014



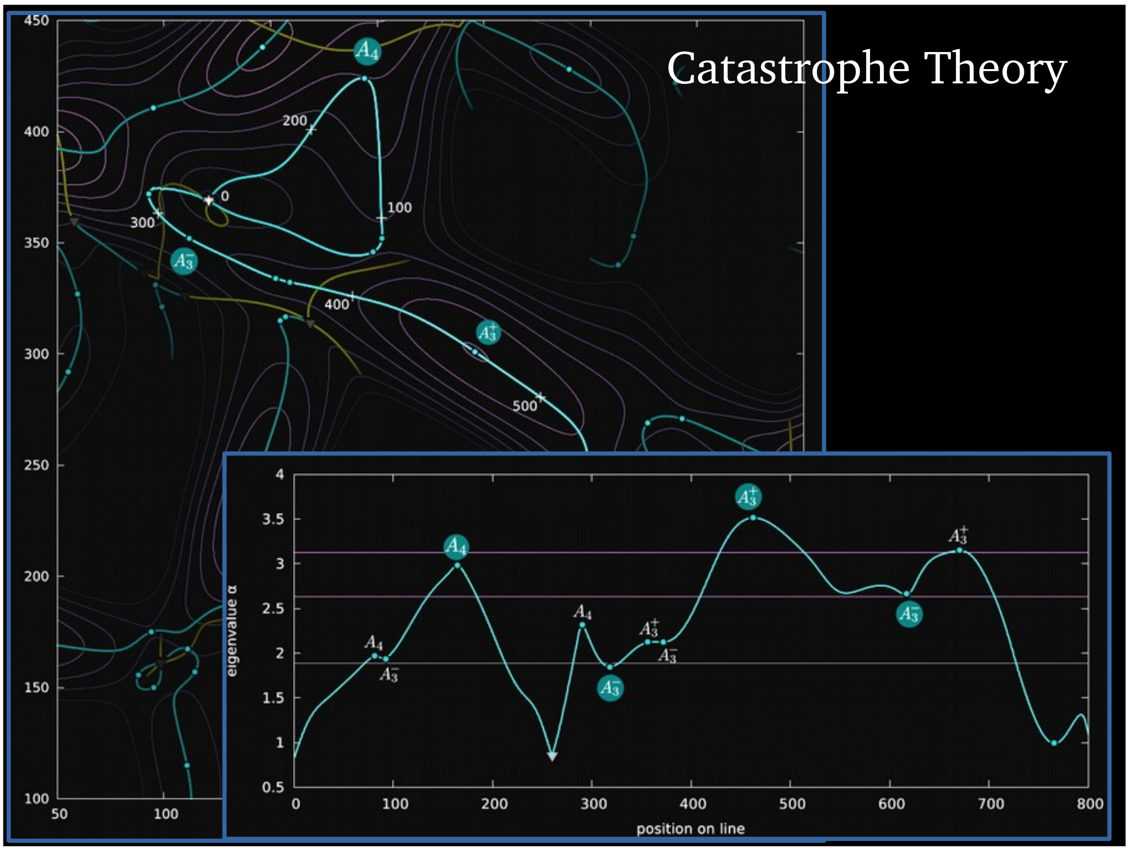
The contours show the level sets of the first eigenvalue. The line elements are the corresponding eigenvectors. Note that these eigenvectors have no direction. This becomes a problem around the D4 singularities. There are two types of D4 singularities (pyramid and purse, or triple and wedge) around which there is no possible consistent vector orientation.

The A3 lines are found where the eigenvector is tangent to the level set of the corresponding eigenvalue. We computed these lines by finding the zero-set of the innerproduct between the gradient of the eigenvalue and the eigenvector. We take special care at the lines where different vector orientations collide.

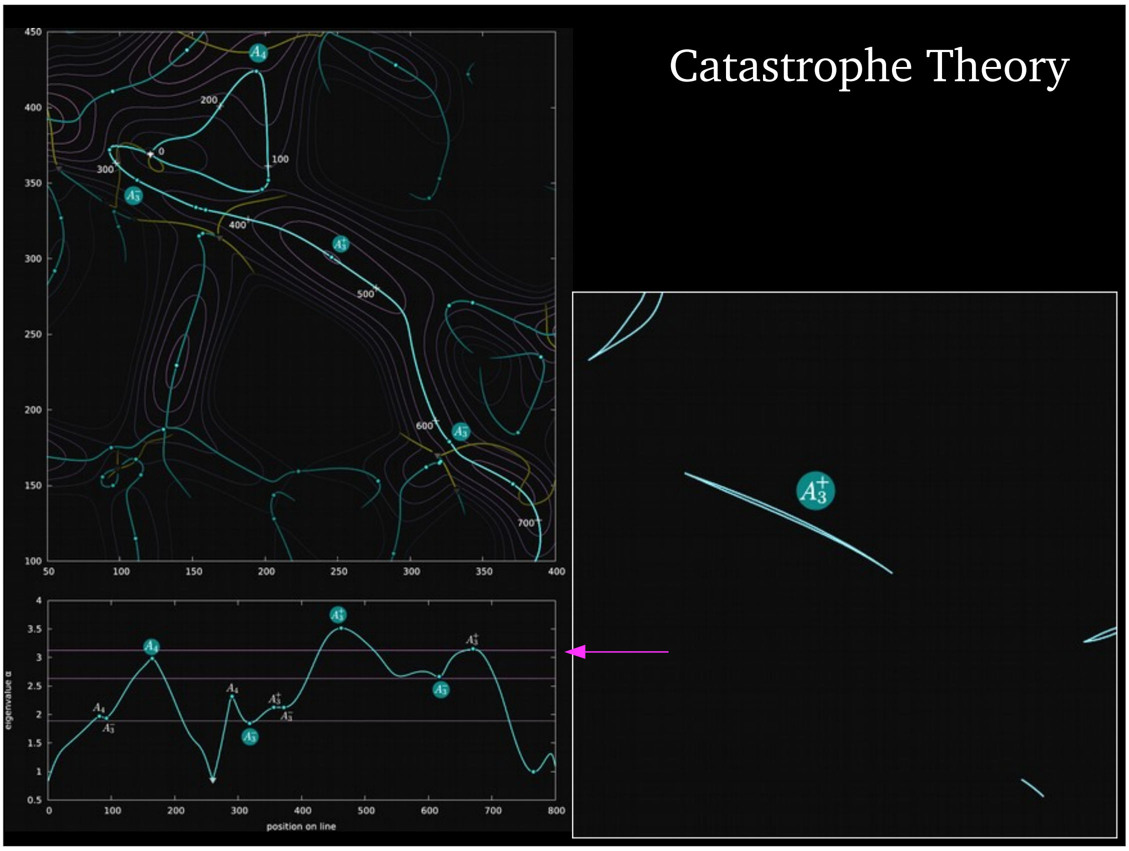
At D4 singularities, the two eigenvalues are equal and a A3 line of one eigenvalue is transformed into an A3 line of the other eigenvalue, either 1 to 1, or 3 to 3.



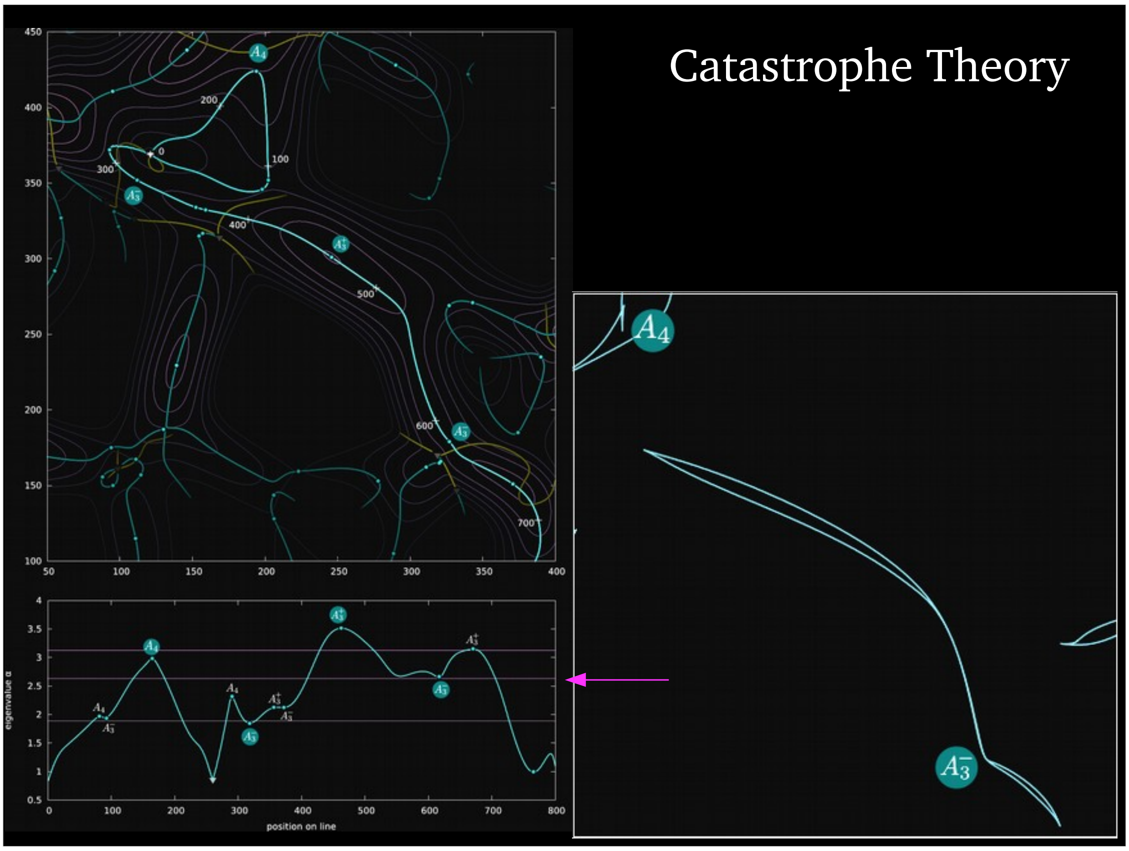
The eigenvalues give a landscape. Note the D4 singularity on the left side of this image.



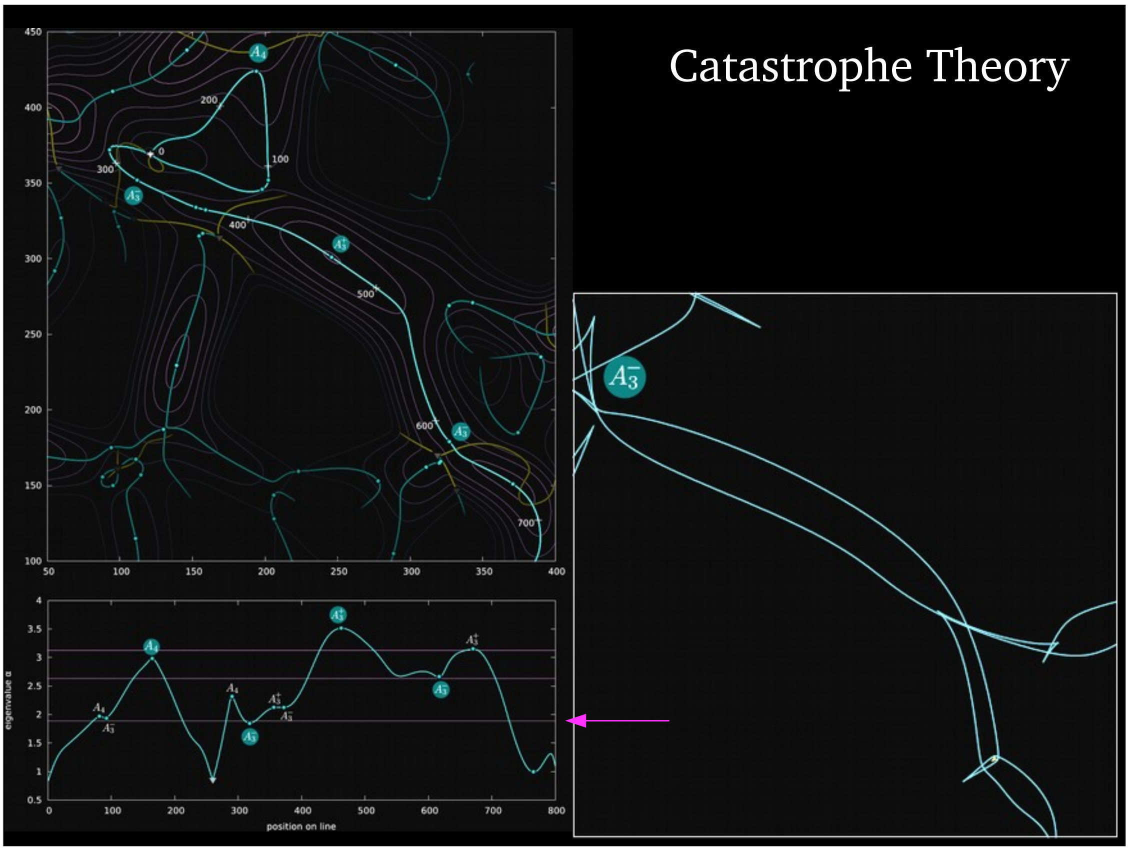
This is the same system from above. I have taken out a filament of which we will follow the formation history. It contains several of the catastrophes that I discussed just a moment ago. The points on this line will go cuspy at time D=1/lambda, which I plotted here as a function of distance on the line. Let's see how this filaments looks in Eulerian space.



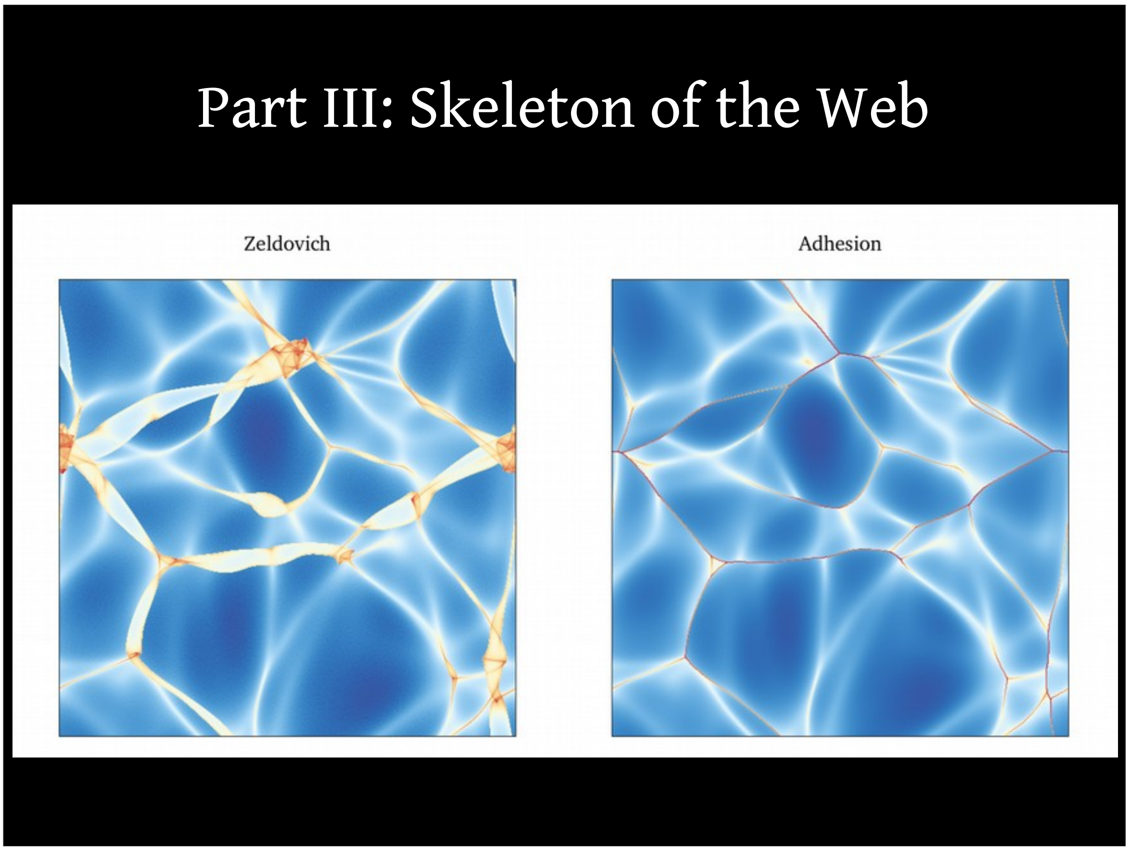
We look at the first image corresponding to the highest purple line in the bottom graph. Along the selected filament, there are two A3+ singularities that signify the birth of a pancake. We have marked the A3 singularity at position 450 and show where it appears in Eulerian space.



Now we look at the second line, just after the A3- near position 600. The A3- singularity is a saddle point of the eigenvalue and signifies the merger of two pancakes.



Finally the filament merges and more complex structures start to appear.



We can also study the skeleton of the multi-stream regions. Certainly when structures are more evolved, the Zeldovich approximation is no longer valid. The adhesion model gives a more accurate description. However, the approaches should be considered complimentary. The Zeldovich approximation tells us what the multi-stream patterns look like, just after the first structures form. The adhesion model tells us how the network evolves in a natural way, but we lose a lot of information on the internal flow of structures and therefore their genesis.

Note that the adhesion formalism gives us the true skeleton of the Zeldovich structures. The topology of the single-stream regions remains the same.

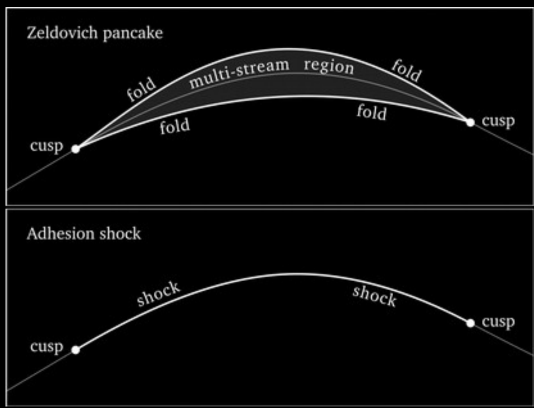
The Adhesion Model

Add viscosity term to Equation of Motion

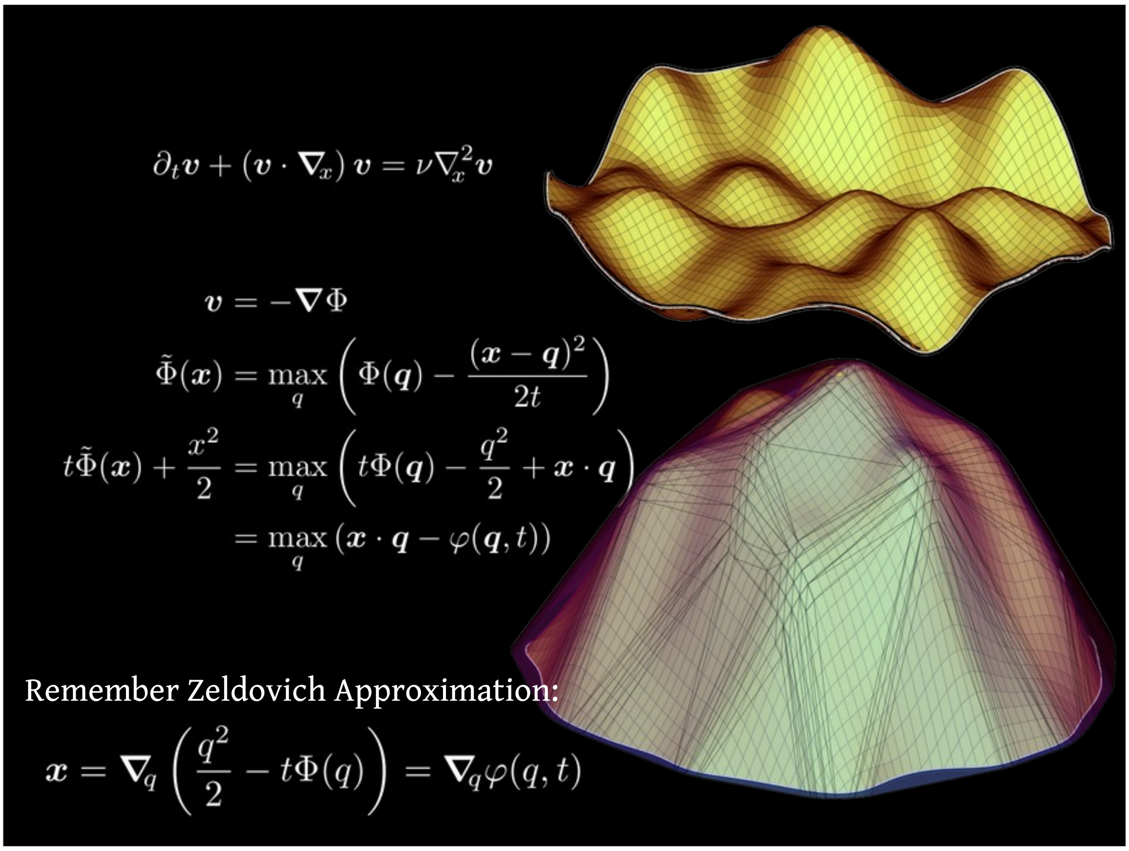
Burgers' equation

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}_x)\, \boldsymbol{v} = \nu \nabla_x^2 \boldsymbol{v}$$

Solved by E. Hopf (1950)

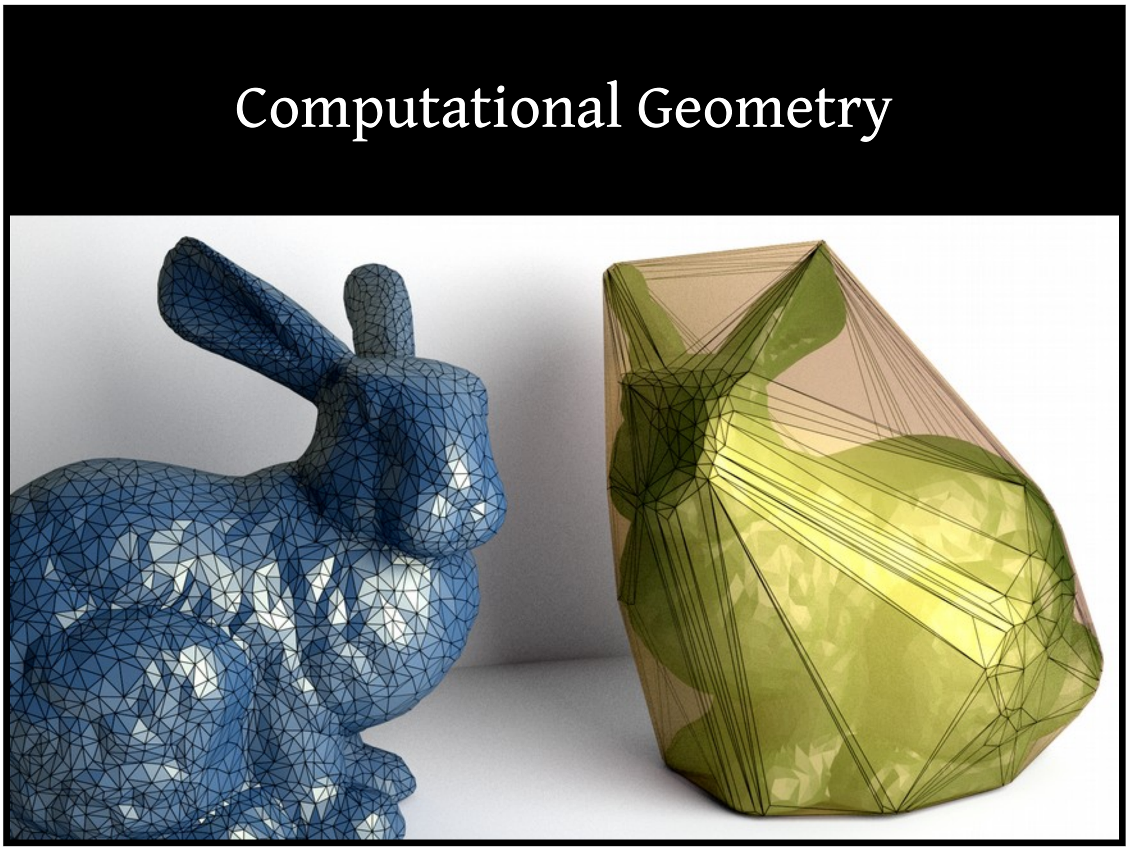


We start from the Zeldovich equation of motion and add a viscosity term. We take the solution where $\nu \to 0$.

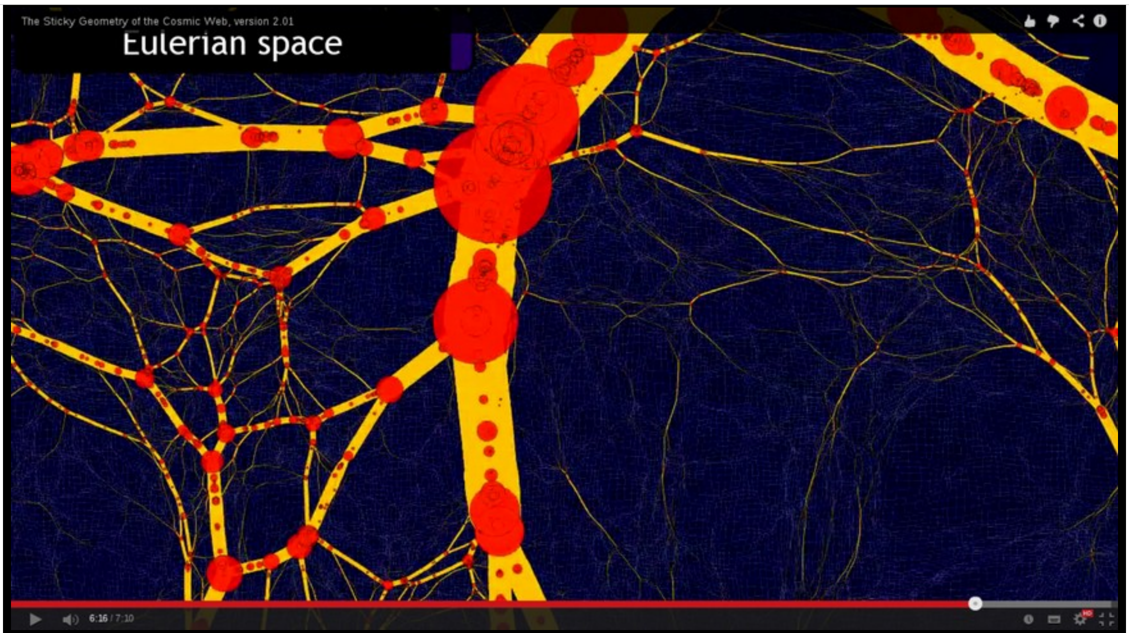


Now the solution is given by the convex hull of the (modified) potential $\varphi = q^2/2 - t\Phi$. This can be understood from the same expression of the Zeldovich Approximation (see expression at the bottom). Where we have multi-stream regions, the Lagrangian map $L{:}q\rightarrow x$ is not monotonic. To make it monotonic we take the convex hull of the generating potential. The resulting shocks in adhesion are the Maxwell strata of the multi-stream regions.

All the singularities that we identified in the Zeldovich Approximation are still relevant in the Adhesion model.

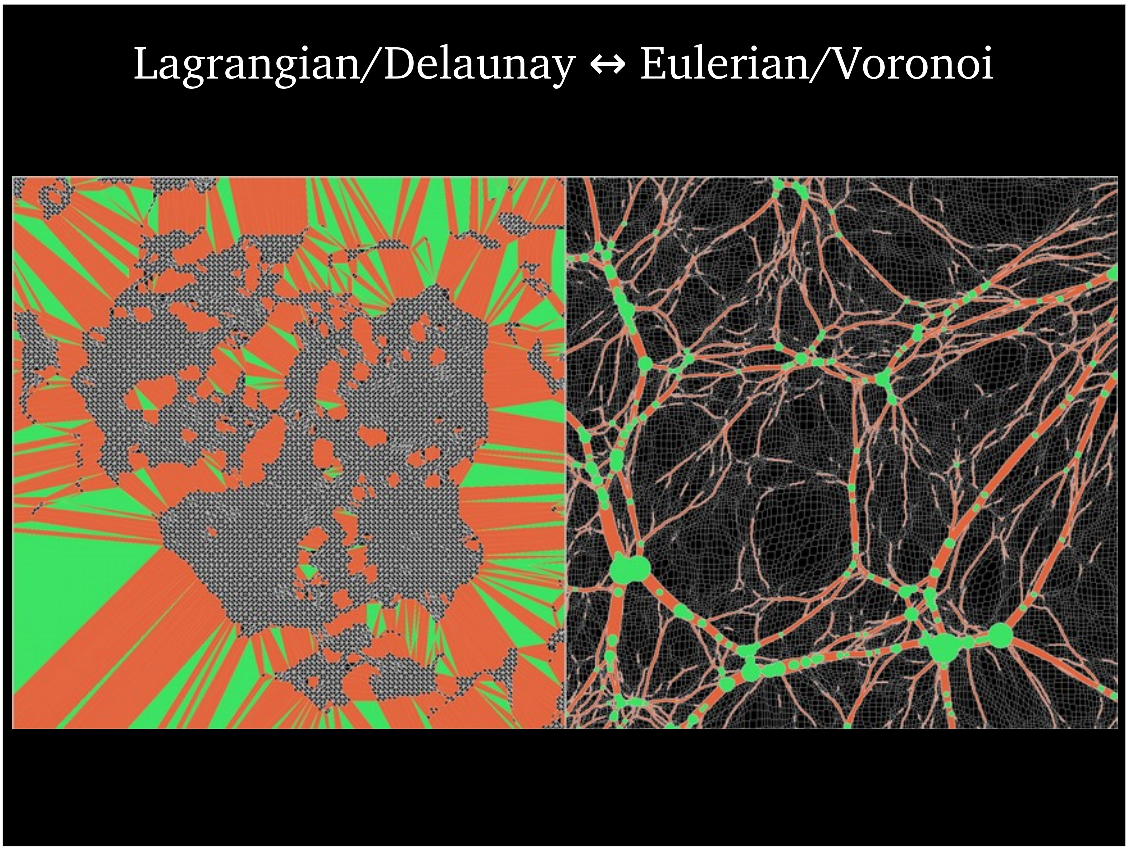


We can now use CGAL to compute the structures resulting from the potential. We use the regular triangulation, weighting a (quasi-)continuous ensemble of points as $w(q) = 2t\Phi(q)$.



http://youtu.be/wI12X2zczqI
(or search: Geometry of Cosmic Web)

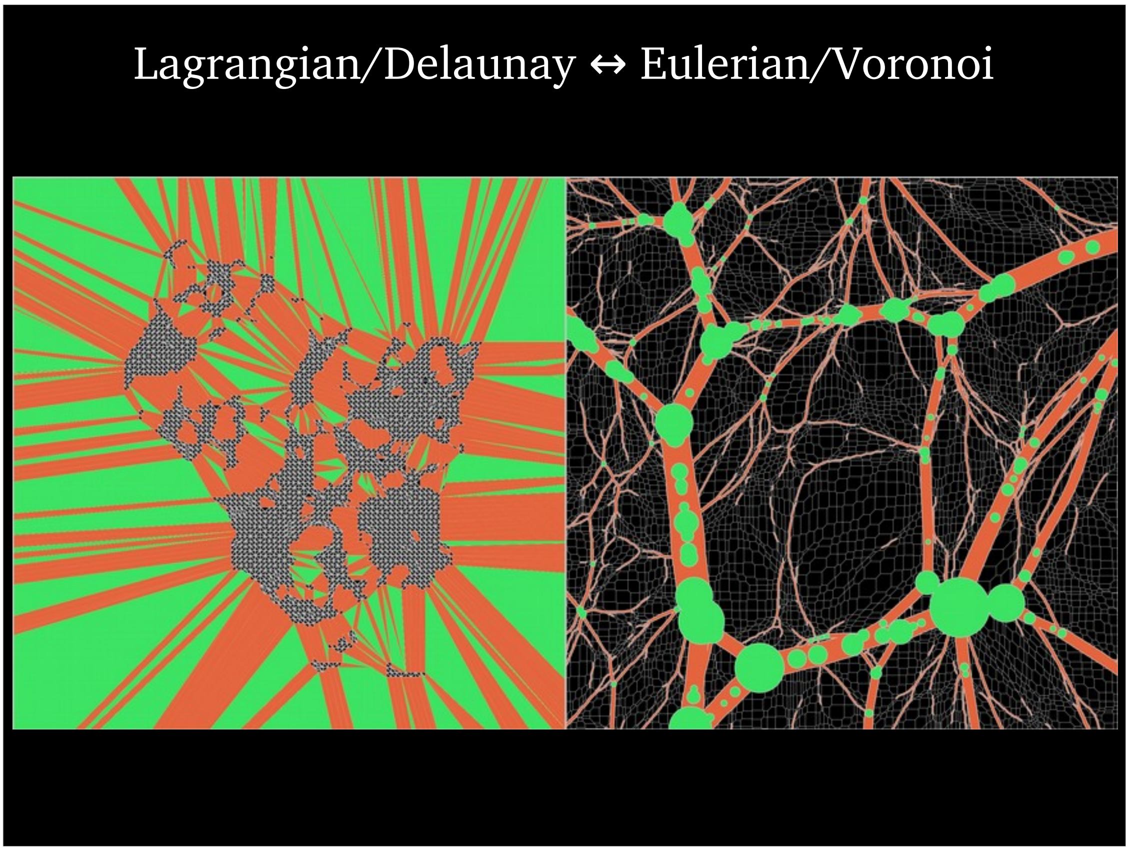
Please look at the movie!



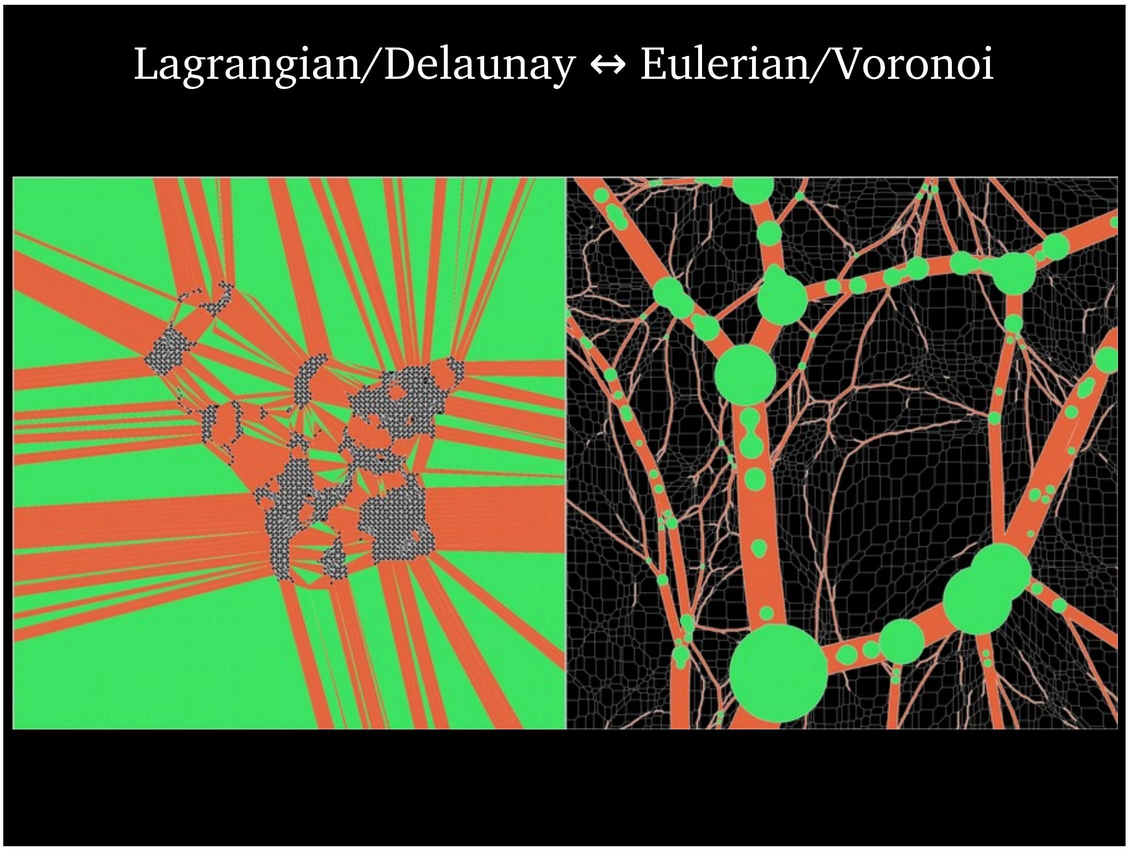
We applied the regular triangulation to a grid of points, weighted with a randomized value. The voids have more power than the surrounding matter. These points remain part of the triangulation. On the other hand, points that collapse into structures become reduntant, adding mass to the structures they represent.

Here we show filaments in orange, their line density scales with the length of the simplex edges.

The nodes are shown in green, their mass given by the area of the delaunay cells.



As time flies, the voids expand in Eulerian space (Voronoi), while losing matter, shrinking in Largrangian space (Delaunay).



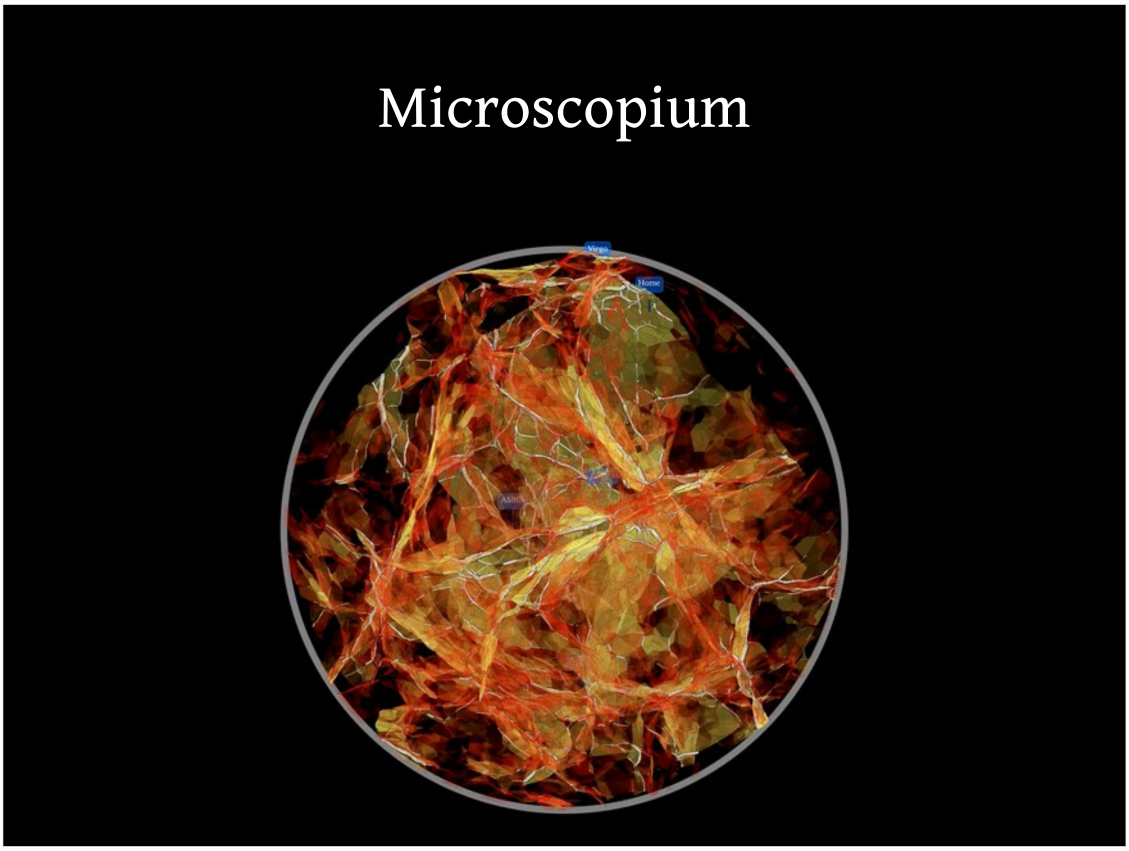
Clusters grow more massive as more vertices become redundant.

Back to *our* Universe

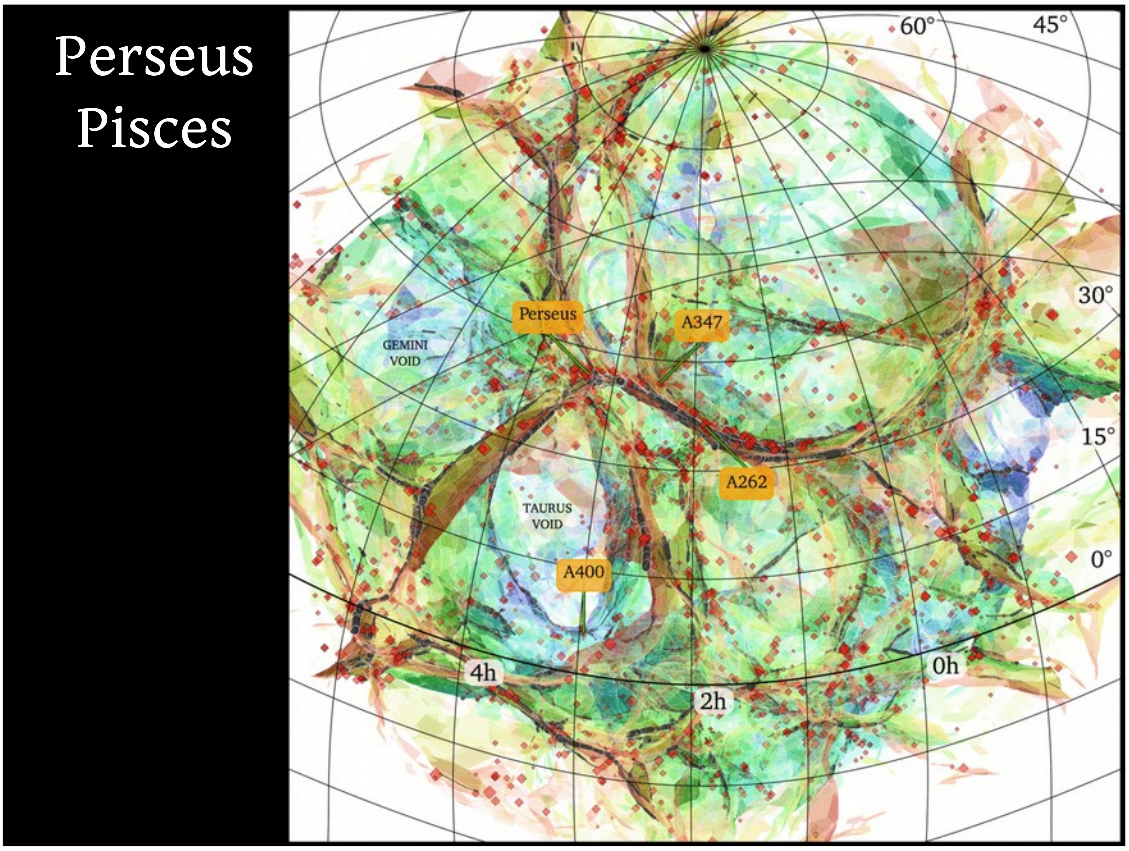
Initial conditions from a *reconstruction* based on galaxies in 2MRS (Kitaura 2012, Heß et al. 2014)

Used CGAL's regular triangulations to find structures

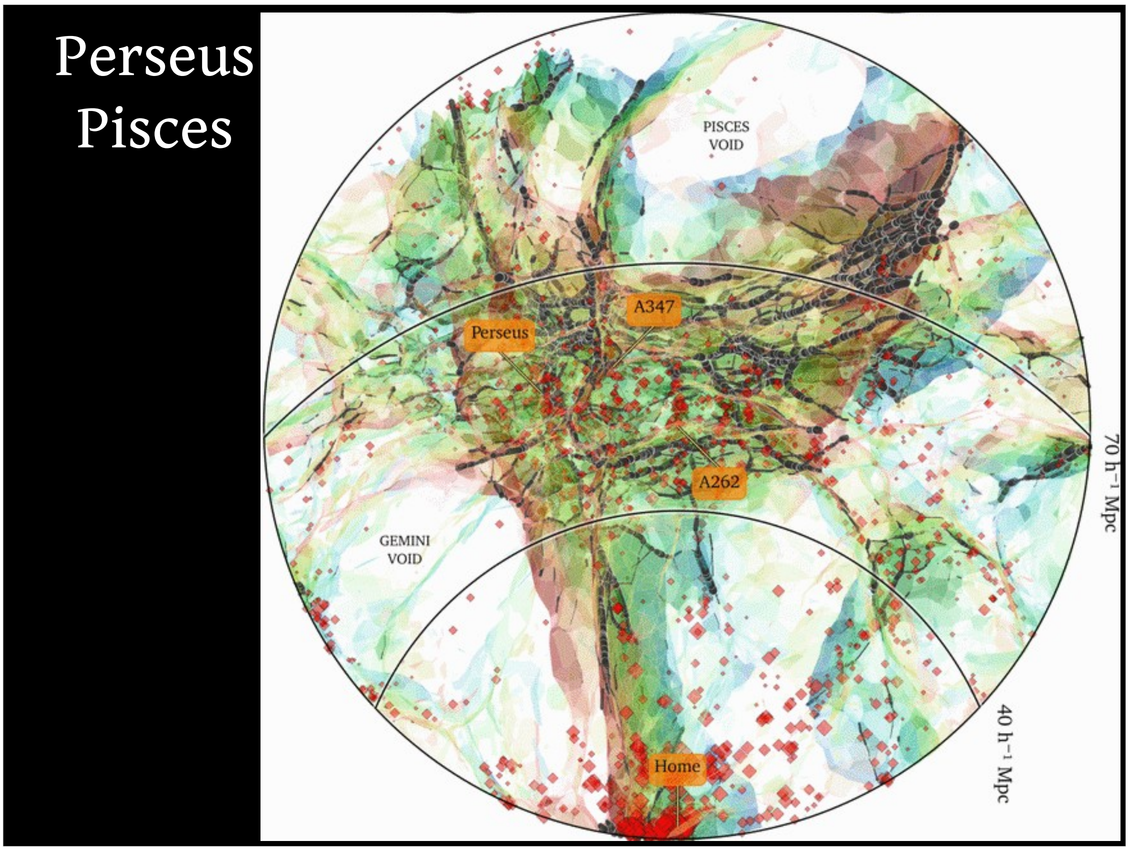
The next pictures and movies are all visualisations of power-diagrams. We used initial condition provided by Heß and Kitaura. They produced initial conditions tailored to produce the structures that we observe in the nearby universe today.



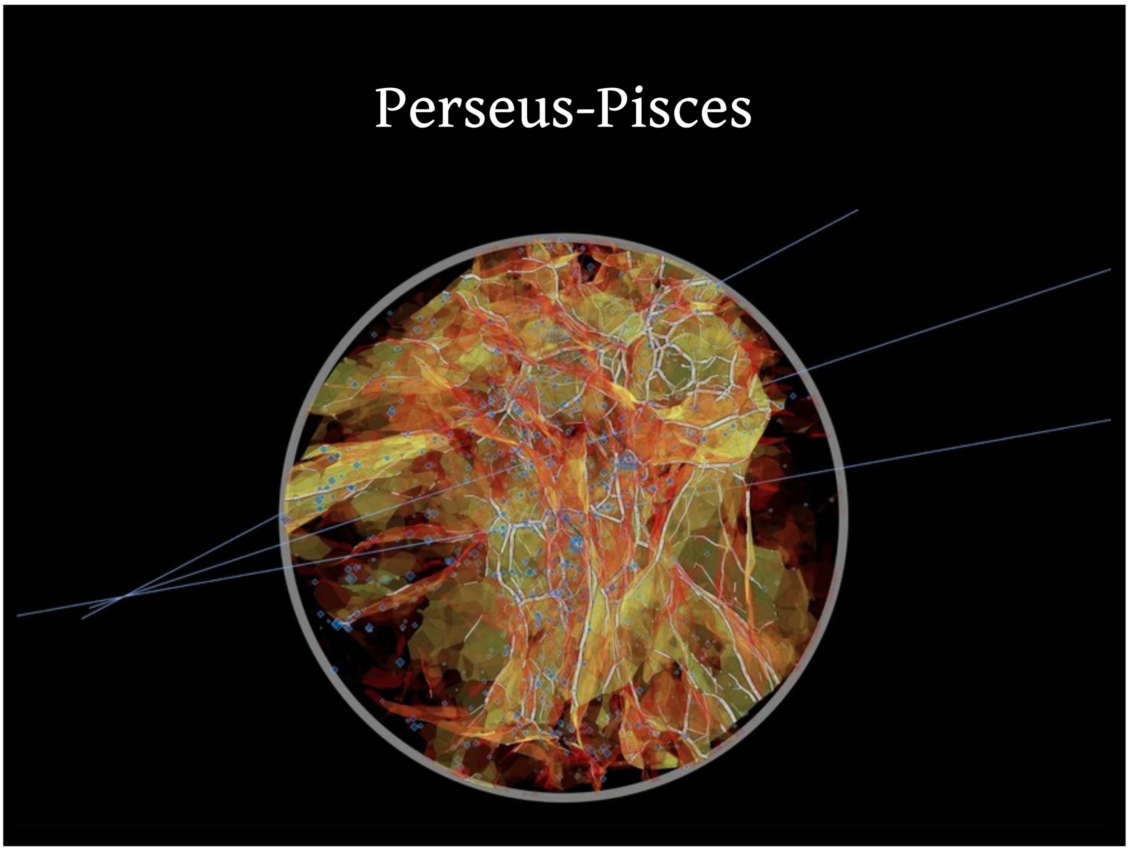
Microscopium is the name of a very very large void right on our doorstep.



Back to the Perseus-Pisces structure. We used the Power diagram to image the strucutres.



This is a top-down view of the same structure, taking a slice along the line Perseus-cluster/A262. The filaments, here in a flattened configuration, are in the process of merging to form a bigger filament.



3D movies galore!